Investigation of the Anomalous Diffusion Coefficients of Different Transport Regimes

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Abstract

The anomalous particle transport induced by magnetic field line structure is studied. Various expressions are known in the literature, which describe the perpendicular diffusion coefficients of different transport regimes. We investigate the problem from a new point of view, using the methods of mixing in chaotic flows. It is shown that the influence of the magnetic properties could be discussed in this general way. New possible regimes are discovered. If we adopt the usual assumptions for the magnetic properties, mainly the parallel diffusion coefficient governs the anomalous perpendicular diffusion.

1. Introduction

Cosmic ray transport normal to the regular component of the Heliospheric Magnetic Field is a complex problem. There is a consensus in the literature that the magnetic field line structure plays an important role in the perpendicular transport. The fluctuating super-Alfvénic flow of the solar wind tangles up the magnetic field lines, which are frozen in the plasma. The resulting complex magnetic background highly influences the motion of the supra-thermal charged particles of relatively low rigidity. The particles generally move along the field lines of this very complex magnetic field structure. Initially neighboring particles can move away in the direction perpendicular to the mean field without real cross-field motion. Various expressions are known describing the perpendicular diffusion coefficients of different transport regimes [3]. It is our purpose here to find a new approach, which allows us to investigate the different regimes in a generalized framework.

2. Mixing

We would like to direct the attention to the similarity of this transport process with the phenomenon of fluid mixing. Mixing is the result of complex interaction between flow and events occurring at small length scales, where the role of the diffusion became important. The particles of an ink-drop diffuse very slowly into the remaining part of the fluid, if the fluid is not stirred. On the

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other hand, the chaotic flow of the fluid deforms the ink-drop into a highly fragmented fractal-like structure; this diffusionless mechanism is called stirring in the literature of chaotic flows. The ink/fluid interface area increases extremely, the concentration gradients become very high and the diffusion, which is very slow unaided, can rapidly flatten out the structure. More precisely this applies only for closed flows without modifications.

In our problem, the field line mixing produces a very complex magnetic structure; field line patches (small areas chosen perpendicular to the mean field) develop into fractal-like configurations if we examine their shape in different points along the mean field direction. Particles, which tend to follow field lines, can move only in these field line bands without perpendicular diffusion. While the particle motion is constrained to the field line bands, this is a complicated non-diffusive motion[2]. The space volume, which the particles can reach increases as $\sqrt{t}$ with time. If a little perpendicular diffusion is possible, the motion is completely different [1,5] the particle motion is diffusive.

Where the characteristic width of the offshoots of patches become so small that the diffusion become important, particles can escape from the field line bands and the complex structure will be flatten out. For large t, the space volume, which the particles can reach increases with time diffusively, the motion can be described with an effective diffusion tensor. The critical parameter in both phenomena is the length scale δ_{eq}, where the effects of stirring and diffusion become roughly equal. The evolution of the field line patch can be treated as a two dimensional mixing, where the distance along the mean field direction plays the role of the time variable of the mixing problem. As it is common in the investigation of the two dimensional chaotic flows, we can further simplify the problem to discover the length δ_{eq}. The local behavior of an offshoot (filament) of the patch can be studied in a reduced one dimensional model. In the presence of chaotic two-dimensional transport one can assign to any points of the flow a convergent and a divergent direction. The patches tend to be elongated exponentially in the divergent direction, and become more and more narrow in the transverse, convergent direction. Gradients are enhanced exponentially in this convergent direction. The stirring process always dominates in the stretching direction, but not in the transverse. Now we are interested in the critical length scale, which is determined by the evolution of the transverse profile. The width (δ) of the filament after a Δt time step can be described as,

$$\delta(t+\Delta t) = F(\Delta s/L)\,\delta(t) + G(\Delta t, \kappa_\perp), \tag{1}$$

where the first term of the right hand side is responsible for stirring, the second for diffusion, and Δs is the average distance, what the particles travel along the mean field direction in a time Δt. It is common to use $F(\Delta s/L) = \exp(-\Delta s/L)$, where the L length scale of the mixing is the reciprocal of the so-called Lyapunov exponent λ. We only use here that the F function can be approximated as

$F(\Delta s/L) \approx 1 - \Delta s/L$. When δ is small enough, the two effects eliminate each other, $\delta(t + \Delta t) = \delta(t) \equiv \delta_{eq}$, and the area of the filament increases exponentially while the envelope area of the field line patch will be smoothly filled with the particles.

3. Diffusion

Many papers using the classical result of Rechester and Rosenbluth [5] define Δs as $\Delta s = v\Delta t$ for collisionless $\Delta s = \sqrt{\kappa_{\|}\Delta t}$ for collisional case; and the function G as $G = \sqrt{\kappa_{\perp}\Delta t}$ for collisionless and $G = \sqrt{\kappa_{\perp}\Delta t} = \Delta s\sqrt{\kappa_{\perp}/\kappa_{\|}}$ for collisional case. There are a few problems with this approach: at first, if the perpendicular scattering events are more rare than the parallel scattering events, why is it possible to describe the parallel motion as scatter free, and the perpendicular as diffusion? Next, these expressions do not include the gradient enhancement in the convergent direction, which makes the diffusion more rapid. The second question answers the first, diffusion is driven by the gradient of the concentration (temperature, etc.) field. If the tangent profile of the field becomes more and more narrow due to the stirring, the gradient, as well as the diffusive broadening of the profile is intensified, which opens the mentioned possibility.

But the definition of Δs and G must be revised in view of the gradient changes. The classical calculations used that well-known solutions of the diffusion equation, in which a quantity of heat/solute is diffusing into a (semi-)infinite body. The diffusive broadening of this solution can be described as $\delta(t) \approx \sqrt{\kappa t}$. Qualitative considerations or a simple derivation gives us that the characteristic width of this solution changes by $\Delta\delta = \frac{\partial}{\partial t}\delta\,\Delta t \approx \frac{\kappa}{\delta}\Delta t$ after a Δt time step. So we have to re-define G as $G = \frac{\kappa_{\perp}}{\delta}\Delta t$ in both collisionless and collisional case, and $\Delta s = \frac{\kappa_{\|}}{l}\Delta t$ for the collisional case. Here l is the extent of the profile parallel to the mean field.

For the collisional case there is another possibility, namely when the particles of our interest are generated by a constant source. In that case the parallel particle motion can be better described asymptotically by a steady flow instead of diffusion. In that case the complex shaped field line band is filled with a steady-state particle cloud, particles diffusively escape through the boundary of the narrow filaments, and the source recovers this loss. $\Delta s = v_{eff}\Delta t$, where v_{eff} is the effective particle speed parallel to the mean field direction. Other, more difficult motions are possible, depending on that which solutions of the diffusion equation can be applied for the parallel and which for the perpendicular transport.

It is easy to calculate using the definition of δ_{eq} that this length scale is, $\delta_{eq} = \sqrt{\kappa_{\perp}/\lambda v} = \sqrt{\kappa_{\perp}L/v}$, which is very similar to the well-known $\sqrt{\kappa_{\perp}/\lambda}$ result of fluid mixing [4], except the anisotropy and presence of v. The speed arise here, because the distance along the mean field direction substitutes the time variable of the mixing problem.

For the collisional, parallel steady flow case the result is similar, but the effective speed v_{eff} arise in the expression.

For the collisional, parallel diffusion case $\delta_{eq} = \sqrt{\kappa_\perp L\, l/\kappa_\parallel} = \sqrt{\frac{\kappa_\perp L}{\kappa_\parallel / l}}$. There is an l dependence in this expression, which means that the equilibrium with of the filaments grows with the parallel distance.

Interestingly, although the microscopic description is significantly different from the classical method, the resulting effective perpendicular diffusion constant is quite similar to the classical result for the formerly known collisionless and collisional parallel diffusion cases of closed flow. The parallel time and length scales of decorrelation (t_d, l) only have logarithmic dependence on δ_{eq}. If we assume that the effective diffusion coefficient is $D_\perp = D_m l/t_d$ [1,3], we get the classical result for the collisionless scale. For the collisional, parallel steady flow case $D_\perp = D_m v_{eff}$. For the parallel diffusion case we can calculate that $l \approx l + 1/2 \ln(l) = L \ln(L_0/\sqrt{L\kappa_\perp/\kappa_\parallel})$, where L_0 is the width of the source. Thus $D_\perp = D_m \kappa_\parallel / L \ln(L_0/\sqrt{L\kappa_\perp/\kappa_\parallel})$. Since there is only logarithmic dependence on $\kappa_\perp$ the parallel perpendicular diffusion dominates the effective perpendicular diffusion as well.

4. Conclusions

The description of charged particle transport induced by the magnetic field line structure is improved using the methods of the chaotic mixing phenomenon. The results depend on that which solutions of the diffusion equation can be applied for the parallel and which for the perpendicular transport. There could be new regimes depending on the properties of particle source. We have briefly investigated the parallel streaming case. Assuming that the field line separation is diffusive we have calculated the effective perpendicular diffusion constant for two classical and one new regimes. Our approach can be applied when the field lines are mixed by an open flow. The particle motion will more complex in such cases, diffusive and non-diffusive regimes are possible.

5. References

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