Probing the Turbulent Solar Wind with Cosmic Rays

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Abstract

Cosmic rays impacting a spacecraft have already passed through and interacted with the turbulent solar wind surrounding the spacecraft; therefore, they carry information on the detailed structure of the turbulence. In particular, the simple unlagged correlation between the magnetic fluctuations and fluctuations of the cosmic ray flux, first discussed by Bieber [1987], can potentially provide unique information on the detailed nature of interplanetary magnetic turbulence. Starting with the Vlasov equation, subject to the usual quasilinear approximations, we derive the leading-order approximation of the particle-field correlation for a general turbulent geometry, including non-axisymmetric turbulence, in a plasma flowing in an arbitrary direction with respect to the average magnetic field.

1. Introduction

Casual inspection of the physical parameters in the interplanetary medium, such as the interplanetary magnetic field (IMF), reveals the existence of statistically significant fluctuations. Studies in solar wind turbulence focus on determining the origin, characteristics, and dynamic evolution of these fluctuations. The statistical moments of the fluctuating quantities, such as the average and the correlation tensor, serve as one of the most important theoretical and observational tools in the analysis of the fluctuating parameters studied in solar wind turbulence.

A spacecraft measuring IMF fluctuations will yield only a one-dimensional, or "reduced", spectrum of the turbulence corresponding to fluctuations sampled along the solar wind flow direction. This reduced spectrum cannot adequately characterize the potentially rich three-dimensional structure of the turbulence. However, energetic particles, such as cosmic rays, impacting the spacecraft have already passed through and interacted with the turbulent field surrounding the spacecraft, and they, in some sense, act as remote probes which carry information on the detailed structure of the turbulence [Bieber, 1990]. In particular, the simple unlagged correlation between the magnetic fluctuations and fluctuations of the cosmic ray flux, $\langle \delta \boldsymbol{B} \, \delta f \rangle$, which is a measurable quantity, can potentially

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provide unique information on the three-dimensional structure of interplanetary magnetic turbulence [Bieber, 1987].

This work, which represents a portion of the Ph.D. thesis of de Koning [2003], extends the theory of Bieber [1987, 1990] by considering the particle-field correlation for a general turbulent geometry in a plasma flowing in an arbitrary direction with respect to the average magnetic field.

2. Quasilinear Theory

The Vlasov equation, subject to the usual quasilinear approximations, returns a formal solution for the the turbulent phase space density,

$$\delta f = \delta f_0 - e \int_0^t \varepsilon^{\imath}{}_{\jmath k}(\bar{t})\, \Delta V^{\jmath}(\bar{\boldsymbol{p}})\, \delta B^k(\bar{\boldsymbol{x}}, \bar{t})\, \partial_{p^{\imath}} \big\langle f(\bar{\boldsymbol{x}}, \bar{\boldsymbol{p}}, \bar{t}) \big\rangle \,\mathrm{d}\bar{t}, \tag{1}$$

where e denotes the electric charge, δB^k denotes the turbulent magnetic field, and $\partial_{p^{\imath}}\langle f\rangle$ denotes the momentum gradient of the average phase space density. In addition, the time dependence of the Levi-Civita tensor, $\varepsilon^{\imath}{}_{\jmath k}$, indicates that the generalized coordinate basis describing the underlying manifold can change from point to point. The term $\Delta V^{\jmath}(\bar{\boldsymbol{p}}) = V^{\jmath}_{SW} - v^{\jmath}$ reflects the role of the flowing plasma in generating particle fluctuations. The solar wind velocity arises from the inclusion of the electric field, which, in an infinitely conducting, flowing plasma, such as the solar wind plasma, is $E^{\imath} = -e\,\varepsilon^{\imath}{}_{\jmath k} V^{\jmath}_{SW} B^k$. When $\delta V_{SW}/V_{SW} \ll \delta B/B$, then $\delta E^{\imath} = -e\,\varepsilon^{\imath}{}_{\jmath k} V^{\jmath}_{SW} \delta B^k$.

To obtain $\langle \delta B^{\hat{\ell}} \delta f \rangle$, multiply both sides of Eq. 1 by $\delta B^{\hat{\ell}}(\boldsymbol{x}, t)$ and ensemble average; when $\delta f_0 = 0$ in all ensemble realizations, this returns

$$\big\langle \delta B^{\hat{\ell}} \delta f \big\rangle = -e \int_0^\infty \varepsilon^{\imath}{}_{\jmath k}(\tau) M^k{}_{\hat{k}}(\tau)\, \Delta V^{\jmath}(\bar{\boldsymbol{p}}) R^{\hat{k}\hat{\ell}}(\boldsymbol{\chi}, \tau)\, \partial_{p^{\imath}} \big\langle f(\bar{\boldsymbol{x}}, \bar{\boldsymbol{p}}, t - \tau) \big\rangle \,\mathrm{d}\tau, \tag{2}$$

where, by definition, the correlation tensor, $R^{\hat{k}\hat{\ell}}(\boldsymbol{\chi}, \tau) \equiv \langle \delta B^{\hat{\ell}}(\boldsymbol{x}, t)\, \delta B^k(\bar{\boldsymbol{x}}, \bar{t}) \rangle$, for $\bar{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{\chi}$ and $\bar{t} = t - \tau$. In addition, $M^k{}_{\hat{k}}$ denotes the transformation matrix from a general coordinate basis to the Cartesian coordinate basis. Notice the change of integrating variable from $\bar{t}$ to τ. Also notice that the upper limit of integration is ∞; this applies to large t when $R^{\hat{k}\hat{\ell}}$ approximately has bounded support on the interval $[-\lambda_C, \lambda_C]$, where λ_C denotes the magnetic correlation length, and $[-\tau_C, \tau_C]$, where τ_C denotes the magnetic decorrelation time.

3. The Particle-Field Correlation

For a pure first-order anisotropy, the average phase space density is

$$\big\langle f \big\rangle = f_0 \Big[1 + \hat{a}_{01}\mu + (1 - \mu^2)^{1/2} (\hat{a}_{11}\cos\phi + \hat{b}_{11}\sin\phi) \Big], \tag{3}$$

where ϕ denotes the particle gyrophase and $\mu = \cos\theta$ denotes the cosine of the particle pitch-angle. Using a pure first-order anisotropy, the leading order particle-field correlation is [see de Koning, 2003, for details on the leading-order approximations of all the terms in Eq. 2]

$$
\begin{aligned}
\tilde{C}^{\hat{s}}_{fB} = &-\hat{a}_{01} e\frac{v}{p}(1-\mu^2)^{1/2}\Big[\mathit{Ct}^{y\hat{s}}(\Omega\tau) - \mathit{St}^{x\hat{s}}(\Omega\tau)\Big] f_0 && \text{(A)}\\
&- eV_{SW}\cos\psi\,(1-\mu^2)^{1/2}\Big[\mathit{Ct}^{y\hat{s}}(\Omega\tau) - \mathit{St}^{x\hat{s}}(\Omega\tau)\Big]\frac{\partial\langle f\rangle}{\partial p} && \text{(B)}\\
&+ eV_{SW}\sin\psi\cos\phi\Big[\mu\,\mathit{Ct}^{y\hat{s}}(0) - (1-\mu^2)^{1/2}\,\mathit{St}^{z\hat{s}}(\Omega\tau)\Big]\frac{\partial\langle f\rangle}{\partial p} && \text{(C)}\\
&+ eV_{SW}\sin\psi\sin\phi\Big[\mu\,\mathit{Ct}^{x\hat{s}}(0) - (1-\mu^2)^{1/2}\,\mathit{Ct}^{z\hat{s}}(\Omega\tau)\Big]\frac{\partial\langle f\rangle}{\partial p} && \text{(C)}\\
&- \Big\{\hat{a}_{11}\mu\Big[\mathit{Ct}^{y\hat{s}}(2\Omega\tau) - \mathit{St}^{x\hat{s}}(2\Omega\tau)\Big] + \hat{b}_{11}\mu\Big[\mathit{Ct}^{x\hat{s}}(2\Omega\tau) + \mathit{St}^{y\hat{s}}(2\Omega\tau)\Big]\\
&+ (1-\mu^2)^{1/2}\Big[\hat{a}_{11}\mathit{St}^{z\hat{s}}(\Omega\tau) - \hat{b}_{11}\mathit{Ct}^{z\hat{s}}(\Omega\tau)\Big]\Big\}\, e\frac{v}{p}\, f_0\,\cos\phi && \text{(4)}\\
&+ \Big\{\hat{a}_{11}\mu\Big[\mathit{Ct}^{x\hat{s}}(2\Omega\tau) + \mathit{St}^{y\hat{s}}(2\Omega\tau)\Big] - \hat{b}_{11}\mu\Big[\mathit{Ct}^{y\hat{s}}(2\Omega\tau) - \mathit{St}^{x\hat{s}}(2\Omega\tau)\Big]\\
&- (1-\mu^2)^{1/2}\Big[\hat{a}_{11}\mathit{Ct}^{z\hat{s}}(\Omega\tau) + \hat{b}_{11}\mathit{St}^{z\hat{s}}(\Omega\tau)\Big]\Big\}\, e\frac{v}{p}\, f_0\,\sin\phi,
\end{aligned}
$$

where $\tilde{C}^{\hat{s}}_{fB} \equiv \mathit{O}^{\hat{s}}{}_{\hat{\ell}}(\phi)\left\langle \delta B^{\hat{\ell}}\,\delta f\right\rangle$ and $\mathit{O}^{\hat{s}}{}_{\hat{\ell}}(\phi)$ denotes a rotation through an angle ϕ counterclockwise about the z-axis. In addition, Ω denotes the particle gyrofrequency. For the sake of brevity, we introduced the turbulence integrals

$$\mathit{Ct}^{\imath\jmath}(\phi) \equiv \int_0^\infty \widetilde{R}^{\hat{\imath}\hat{\jmath}}(\tilde{\boldsymbol{\chi}},\tau)\,\cos\phi\,\mathrm{d}\tau, \tag{5-1}$$

$$\mathit{St}^{\imath\jmath}(\phi) \equiv \int_0^\infty \widetilde{R}^{\hat{\imath}\hat{\jmath}}(\tilde{\boldsymbol{\chi}},\tau)\,\sin\phi\,\mathrm{d}\tau, \tag{5-2}$$

where $\widetilde{R}^{\hat{q}\hat{s}}(\tilde{\boldsymbol{\chi}},\tau) = \mathit{O}^{\hat{q}}{}_{\hat{k}}(\phi)R^{\hat{k}\hat{\ell}}(\boldsymbol{\chi},\tau)\mathit{O}^{\dagger}{}_{\hat{\ell}}{}^{\hat{s}}(\phi)$ and $\tilde{\chi}^{\hat{a}} = \mathit{O}^{\hat{a}}{}_{\hat{b}}(\phi)\chi^{\hat{b}}$. More explicitly, the rotated lagged position is

$$\tilde{\chi}^x = R_L\sin\theta\sin\Omega\tau + \tau V_{SW}\sin\psi\cos\phi, \tag{6-1}$$

$$\tilde{\chi}^y = R_L\sin\theta\big(1-\cos\Omega\tau\big) - \tau V_{SW}\sin\psi\sin\phi, \tag{6-2}$$

$$\tilde{\chi}^z = \mu v\tau. \tag{6-3}$$

Equation 4 expresses the central result of this work; this equation is the leading order expression of the particle-field correlation for completely general homogeneous and stationary turbulence geometry and a pure first order cosmic ray anisotropy in a flowing plasma. Equation 4 applies to simple turbulent geometries, such as slab turbulence, as well as completely general geometries, such as non-axisymmetric turbulence.

Notice the symmetry between the three components of the particle-field correlation. For $\hat{s} = (x, y, z)$, line A expresses the solar wind independent gyrotropic component of the particle-field correlation; line B expresses the solar wind dependent gyrotropic component; line C expresses the solar wind dependent gyro-anisotropic component; and the unmarked lines express the solar wind independent gyro-anisotropic component. Notice that the solar wind independent gyrotropic component, line A, depends on the field-aligned anisotropy, $\hat{a}_{01}$, while the solar wind independent gyro-anisotropic terms depend on the non-field-aligned anisotropy, $\hat{a}_{11}$ and $\hat{b}_{11}$. Furthermore, notice that in the solar wind independent gyro-anisotropic terms, some of the turbulence integrals are evaluated at 2Ω, whereas in the solar wind dependent gyro-anisotropic terms, some of the turbulence integrals are evaluated at zero frequency. Finally, notice that the z-component of the particle-field correlation, $\tilde{C}^{\hat{3}}_{fB} = \langle \delta B^z \delta f \rangle$, only depends on $R^{z\hat{s}}$ and $R^{\hat{s}z}$.

In summary, the rotated particle-field correlation has the form,

$$\tilde{C}^{\hat{\imath}}_{fB} = c^{\hat{\imath}}_g(p,\mu) - c^{\hat{\imath}}_c(p,\mu)\cos\phi - c^{\hat{\imath}}_s(p,\mu)\sin\phi. \tag{7}$$

which clearly expresses the gyrophase dependence of the rotated particle-field correlation.

References

1. J. W. Bieber. *20th ICRC*, vol. 3, p. 189, 1987.
2. J. W. Bieber. *21st ICRC*, vol. 5, p. 308, 1990.
3. C. A. de Koning. PhD thesis, University of Delaware, 2003.

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