Pitch angle diffusion of energetic particles by large amplitude MHD waves

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Abstract

We discuss some fundamental properties of pitch angle diffusion of charged particles by MHD waves by performing test particle simulations. Even at a moderate normalized turbulence level (turbulence magnetic field energy density normalized to the background field energy density ~ 0.1), both the mirroring and the resonance broadening effects become important, and the diffusion starts to deviate substantially from the standard quasi-linear diffusion model. Some other outstanding consequences of the finite amplitude of the waves are the dependence of the diffusion coefficients on spatial and temporal scales, and the effect of phase coherence among MHD waves, as evidenced by recent spacecraft data analysis.

1. Introduction

Transport of energetic particles (cosmic rays) by MHD turbulence is one of the key issues in space and astro-plasma physics. Pitch angle diffusion is fundamental to other transport processes such as the energy and the parallel diffusion[4,9,10,11]. For the discussion of the various transport processes, the quasi-linear theory is frequently used, in which two assumptions are fundamental. First, the turbulence amplitude is sufficiently small, so that truncation at the second power of the turbulence is guaranteed. Second, the wave phases are random (random phase approximation), so that any effect of mode-mode coherence is destroyed by phase mixing. However, the MHD turbulence in space does not necessarily satisfy these assumptions: in particular, the waves excited near collisionless shocks have the wave magnetic field amplitude comparable or even larger than the background field. Also, their waveforms show consequences of strong nonlinear evolution (e.g., the shocklets found in the earth's foreshock region [3]), suggesting the presence of the phase coherence [2]. From this viewpoint, we discuss the pitch angle diffusion of energetic particles by MHD waves, which are not necessarily small amplitude, and their phases not necessarily random, by numericallly integrating in time the equations of motion of charged particles under influence of given MHD turbulence.

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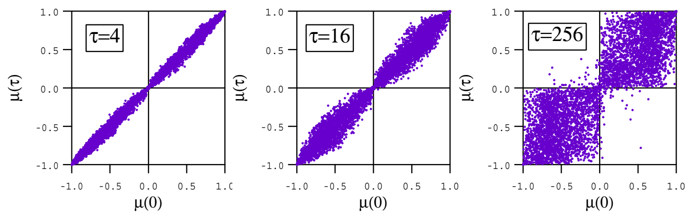

Fig. 1. Time evolution of μ for $\delta B = 0.01$.

2. Model

We employ the so-called slab model for the MHD turbulence, although this is probably an over-simplification for the turbulence in reality (e.g., in the solar wind [8]). Within this model, the fluctuation electromagnetic field is given as a superposition of parallel propagating, circularly polarized finite amplitude Alfven waves, with different wave numbers and different polarizations. Since the typical particle velocity far exceeds the Alfven wave speed, we let the waves to be non-propagating: within this system, particle energy is conserved. For both groups of waves with different polarizations, we assume that the wave spectrum is given by a power law (with an index γ), and their phases be related by the iteration formula defined in (4) of [6].

3. Results

Figure 1 shows the time evolution of distribution of particle pitch angle cosine, μ, defined as an inner product of the unit vectors parallel to the particle velocity and the local magnetic field. For each panel, the horizontal axis represents the initial distribution, $\mu(0)$, and the vertical axis denotes the distribution at some later times, $\mu(\tau)$. Each dot represents a single test particle. Important parameters used here are: $\gamma = 1.5$, $c_\phi = 0$ (random phase), and the variance of the normalized perpendicular magnetic field fluctuations, $\delta B = 0.01$. At $\tau = 1$, the distribution

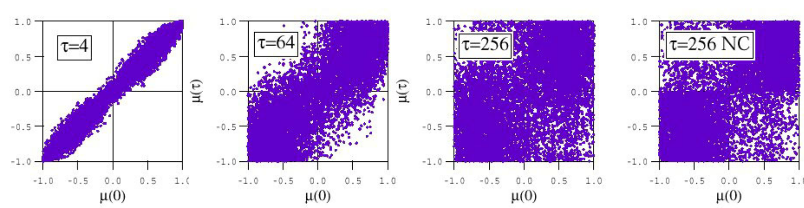

Fig. 2. Time evolution of μ for $\delta B = 0.1$. Non-compressional turbulence is used for the run shown in the right most panel.

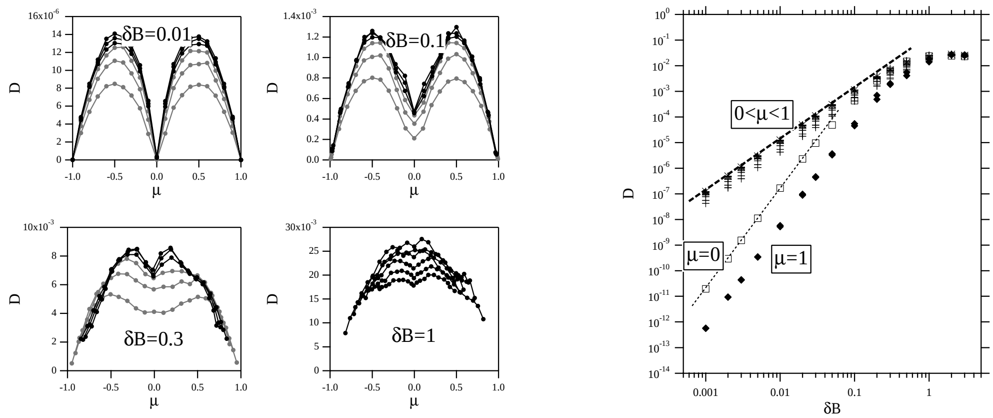

Fig. 3. Right panels: D versus μ. Left panel: D versus δB.

of μ has not evolved much, and so the dots are almost aligned along the diagonal line. Later at $\tau = 16$, pitch angle diffusion is more evident, but is still absent around $\mu \sim 0$ and $|\mu| \sim 1$. The former is due to the lack of waves which resonae with near 90 degrees pitch angle, and the latter is simply due to geometry. Even later time at $\tau = 256$, substantially longer than the pitch angle diffusion time scale, it is clear that the majority of particles stay within the hemisphere they belonged to initially.

Three panels from the left in Figure 2 show the same plots as before except that the turbulence level is increased to $\delta B = 0.1$, keeping other parameters unchanged. From the comparison of the two runs it is clear that not only the diffusion occurs at a faster time scale but also that many particles traverse the 90 degree pitch angle. This is mainly due to the mirroring and the resonance broadening, both of which are the consequences of finite amplitude waves. We can separate these two effects by making the turbulence non-compressional, $\mathbf{b}'(x) = \delta B \ \mathbf{b}(x)/|\mathbf{b}(x)|$, where $\mathbf{b}(x)$ is the given compressional turbulence (the power spectrum and the phase distribution of $\mathbf{b}(x)$ and $\mathbf{b}'(x)$ are not exactly the same). The distribution of μ as diffused by such a non-compressional turbulence is shown in the right most panel of Figure 2. Although the number of particles crossing the 90 degrees pitch angle is less compared with the compressional case, it is shown that the resonance broadening alone can mix the particles across $\mu = 0$.

Figure 3 summarizes the numerically evaluated pitch angle diffusion coefficient, D, compared with the value obtained from the quasi-linear theory,

$$D_{QL} = \frac{\pi e^2}{2m^2c^2v|\mu|}(1-\mu^2)P(k_r), \tag{1}$$

where $k_r = -\Omega/v\mu$ is the resonance wave number, $P(k)$ is the wave power spectrum, Ω is the particle gyro-frequency, and other notations are standard [1,5,7]. Four panels in the left show D versus μ, for various values of δB. The turbulence is compressional, and the wave phases are random. When δB is small, D is doubly peaked, as it vanishes at $\mu = 0$, -1, and 1. However, as the turbulence amplitude is increased, the diffusion at $\mu = 0$ becomes drastically enhanced. At $\delta B \sim 0.3$, D is of the same order with respect to μ. This is also apparent in the right panel of Figure 3, in which D is plotted against δB. When $0 < \mu < 1$, numerically computed D matches well with D_{QL} (thick broken line), while they start to deviate around $\delta B \sim 0.1$.

4. Discussion

From the numerical results obtained above, we are tempted to model the pitch angle diffusion process by a simple equation,

$$\frac{\partial f(\mu)}{\partial t} = \frac{\partial}{\partial \mu} D^* \frac{\partial f(\mu)}{\partial \mu} - \frac{f(\mu) - f(-\mu)}{\tau(\mu)}, \tag{2}$$

where $f(\mu)$ is the distribution function, $D^*(\mu)$ is the (modified) pitch-angle diffusion coefficient including the resonance broadening effect (and thus $D^*(0) \neq 0$), and $\tau(\mu)$ is the time scale for the mirror reflection, which may be determined by statistics of compressional magnetic field (one should note, however, that the mirror reflection is not always addiabatic as assumed in (2)). If there is a finite coherence in the MHD turbulence, as evidenced by recent spacecraft data analysis[2], it strongly influences $\tau(\mu)$, which in turn modifies the pitch angle diffusion.

5. References

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