Finite-Time Shock Acceleration

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Abstract

Observations of energetic ion acceleration at interplanetary shocks sometimes indicate a spectral rollover at $\sim$ 0.1 to 1 MeV/nucleon. This rollover is not well explained by finite shock width or thickness effects. At the same time, a typical timescale of diffusive shock acceleration is several days, implying that the process of shock acceleration at an interplanetary shock near Earth usually gives only a mild increase in energy to an existing seed particle population. This is consistent with recent analyses of *ACE* observations that argue for a seed population at substantially higher energies than the solar wind. Therefore an explanation of typical spectra of interplanetary shock-accelerated ions requires a theory of finite-time shock acceleration, which for long times (or an unusually fast acceleration timescale) tends to the steady-state result of a power-law spectrum. We present analytic and numerical models of finite-time shock acceleration. For a given injection momentum p_0, after a very short time there is only a small boost in momentum, at intermediate times the spectrum is a power law with a hump and steep cutoff at a critical momentum, and at longer times the critical momentum increases and the spectrum approaches the steady-state power law. The composition dependence of the critical momentum is different from that obtained for other cutoff mechanisms.

1. Introduction

The spectral form of [3], a power law in momentum with an exponential rollover in energy, has proven very useful in fitting spectra of solar energetic particles. The composition dependence of the rollover energy depends on the physical effect that causes the rollover.

For traveling interplanetary shocks well outside the solar corona, observations typically indicate a rollover at $\sim$ 0.1 to 1 MeV/n. Let us consider what physical mechanism could explain this. If there is a cutoff for κ/u on the order of the shock thickness [3], where κ is the parallel diffusion coefficient and u is the fluid velocity along the field, the observed long mean free paths for pickup ions [4] would imply an extremely low cutoff energy. On the other hand, a cutoff due to

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shock-drift acceleration across the entire width of a shock (such as that inferred for anomalous cosmic rays) is on the order of hundreds of MeV per charge unit.

We propose that the physical origin of such rollovers is the finite time available for shock acceleration. The typical acceleration timescale t_{acc} corresponding to observed mean free paths is on the order of several days, so the process of shock acceleration at an interplanetary shock near Earth should usually give only a mild increase in energy to an existing seed particle population. Indeed, recent analyses of *ACE* observations argue for a seed population at substantially higher energies than the solar wind [1]. On the other hand, finite-time shock acceleration should yield the standard power-law spectrum in the limit of a long duration t relative to the acceleration timescale. As a corollary of this idea, for an unusually strong shock (unusually short acceleration timescale) it is possible to obtain power-law spectra up to high energies (e.g., as observed by [5]). Therefore, the present work derives a simple theory of finite-time shock acceleration and explores implications for the composition dependence of the spectrum.

2. Analytical and Numerical Models

Consider a combinatorial model of finite-time shock acceleration assuming a constant acceleration rate r (i.e., the rate of a complete cycle returning upstream, or $1/\Delta t$ of [2]) and a constant escape rate ϵ. After a time t, the distribution of residence time T is

$$P(T) = \epsilon e^{-\epsilon T} + e^{-\epsilon t}\delta(T - t). \tag{1}$$

The Poisson distribution of the number of acceleration events n during T is

$$P(n, T) = \frac{(rT)^n}{n!} e^{-rT}. \tag{2}$$

The overall probability of n acceleration events is

$$\begin{aligned} P(n,t) &= \int_0^t P(n,T)P(T)dT \qquad (3) \\ &= \frac{\epsilon}{r}\left(\frac{r}{r+\epsilon}\right)^{n+1} e^{-(r+\epsilon)t} \sum_{k=n+1}^{\infty} \frac{[(r+\epsilon)t]^k}{k!} + \quad e^{-(r+\epsilon)t}\frac{(rt)^n}{n!}. \qquad (4) \end{aligned}$$

Note that the first term is an exponential in n times a Poisson probability of $> n$ acceleration events, and the second term, corresponding to a finite probability of residence time $T = t$, is a Poisson distribution at $\langle n \rangle = rt$. Usually $\epsilon \ll r$ so the result (in terms of momentum) is a power law spectrum with a hump and subsequent cutoff after $\sim rt$ acceleration events. A more complicated analytic expression can be derived for the more realistic case where r and ϵ depend on n (and particle momentum).

The following system of differential equations can be shown to be equivalent to the above approach, and is more convenient for computations. We express $P(n,t)$ as the sum of $E(n,t)$ and $A(n,t)$, the fraction of particles escaping and remaining, respectively, after n acceleration events at time t. Then

$$\frac{dA_n}{dt} = -(r_n + \epsilon_n)A_n + r_{n-1}A_{n-1} \qquad \frac{dE_n}{dt} = \epsilon_{n-1}A_{n-1} \tag{5}$$

with the initial condition $A(0,0) = 1$ and all other A, E zero at $t = 0$.

For a general shock angle, we use r_n and ϵ_n that depend on the particle velocity v_n (following [2]):

$$r_n = \frac{v_n/4}{\frac{\kappa_1}{u_1} + \frac{\kappa_2}{u_2}}, \qquad \epsilon_n = \frac{u_2 \cos\theta_2 / \cos\theta_1}{\left(\frac{\kappa_1}{u_1} + \frac{\kappa_2}{u_2}\right)\left(1 - \frac{4u_2\cos\theta_2}{v_n\cos\theta_1}\right)}, \tag{6}$$

where θ is the field-shock normal angle, the subscript 1 refers to upstream of the shock, and 2 refers to downstream. The particle momentum increases at each acceleration event according to

$$\frac{p_n}{p_{n-1}} = 1 + \frac{4}{3}\frac{u_1\cos\theta_1 - u_2\cos\theta_2}{v_{n-1}\cos\theta_1}. \tag{7}$$

The differential energy spectrum vs. kinetic energy T is calculated from $j(T,t) = P(n,t)/(T_{n+1} - T_n)$.

3. Results

Figure 1 shows results for the time-dependent energy spectrum of ^{4}He for an oblique shock with $u_1 = 540$ km/s, $u_2 = 140$ km/s, $\theta_1 = 45°$, $\theta_2 = 75.5°$, $\kappa = v\lambda/3$, and a parallel scattering mean free path $\lambda = 0.3$ AU (based on Rankine-Hugoniot conditions [6]). Note that this corresponds to injection at 0.01 MeV/n; for an interplanetary shock, the resulting spectrum would be the convolution of such a "kernel" with the seed particle spectrum.

We see that after a short time the particles receive only a small boost in energy. At intermediate times, there is a power law at low energy and a hump at a certain critical energy, T_c, followed by a drastic decline. The power law and hump correspond to the two terms on the right hand side of (1); in particular, the hump corresponds to the fraction of particles that have not yet escaped and have a Poisson distribution of acceleration events n, with $\langle n \rangle \approx rt$. It is not clear whether a hump would be expected in observations, after convolution with the seed spectrum. The decline at high energy is qualitatively similar to that of [3]; however, we obtain a different (Q/A) dependence, as shown below. At very long times, the classic steady-state power law is recovered.

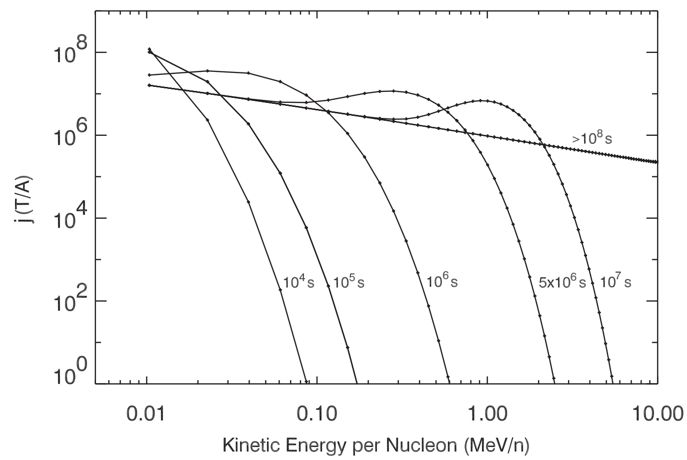

Fig. 1. Energy spectrum of ^{4}He, injected at 0.01 MeV/n, after shock acceleration for the indicated times.

For a constant acceleration rate r, i.e., a constant λ, we expect the critical rigidity P_c to be approximately P_n for $n = rt$:

$$P_c \approx P_0 + \frac{4}{3}\frac{u_1\cos\theta_1 - u_2\cos\theta_2}{\cos\theta_1}\frac{mc}{q}rt \tag{8}$$

$$\approx P_0 + \frac{A}{Q}\frac{m_p c}{e}\frac{u_1\cos\theta_1 - u_2\cos\theta_2}{\cos\theta_1}\frac{1}{\frac{\lambda_1}{u_1}+\frac{\lambda_2}{u_2}}t \tag{9}$$

(assuming non-relativistic particles). Thus for small P_0 or long times, we expect the rollover rigidity to increase proportionally with time (with only a weak dependence on P_0), and the rollover energy to increase as t^2.

Note that for the above case of constant λ the rollover velocity (v_c) and kinetic energy per nucleon (T_c/A) are independent of Q/A. For the more general case of $\lambda \propto P^\alpha$ it can be shown that

$$P_c = \left[P_0^{\alpha+1} + \frac{4}{3}(\alpha+1)\frac{A}{Q}\frac{m_p c}{e}\frac{u_1\cos\theta_1 - u_2\cos\theta_2}{\cos\theta_1}r_0 P_0^\alpha t\right]^{1/(\alpha+1)}, \tag{10}$$

or for late times, $T_c/A \propto (Q/A)^{2\alpha/(\alpha+1)}$. For example, if $\lambda \propto P^{1/3}$ then $T_c/A \propto (Q/A)^{1/2}$, a somewhat weaker dependence than the proportionality to (Q/A) that is sometimes assumed.

References

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