Statistical Procedure to Test Significance in the Analysis of Cosmic Ray Data by Superposed Epoch Method–II

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Abstract

This paper describes a technique to test significance level of the results obtained on the basis of superposed epoch analysis. The test procedure is based on F-test. Details of the procedure to test the Forbush-decrease effect in cosmic ray intensity are discussed. An appropriate procedure is described for the transformation of the data (removal of solar cycle effect). Although this technique is applicable to solar/ astrophysical/ heliospheric/ magnetospheric/ ionospheric/ atmospheric/ meteorological data, the procedure is illustrated using cosmic ray data.

1. Introduction

Superposed epoch analysis has been employed very profitably for many space physics problems. Although this method of analysis is being used since several decades, a suitable procedure to determine the statistical significance of the results is still required. Forbush and coworkers [2] developed a statistical procedure for evaluating the quasi-persistency in the data analyzed by superposed epoch method. Test of significance procedure for any deviation (effect) in the parameters under study, based on Chree analysis, have been given earlier too [6,7]. But, rarely any of these have been exploited in subsequent work. We suspect that (superimposed) solar cycle effect observed in most of the heliospheric/ magnetospheric/ ionospheric/ atmospheric data, some times even larger than the 'signal' amplitude, may even lead to unexpected conclusion and hamper the effort.

2. Method

Let X_{ij} be the random variable that denotes the j^{th} measurement taken from i^{th} population. Let J denotes the number of the observations in the each sample (epoch). I is the number of populations (epochs) being analyzed. Let individual sample means (row average) be $\bar{X}_{1.}$,$\bar{X}_{2.}$, ..., $\bar{X}_{i.}$, and column averages $\bar{X}_{.1}$,$\bar{X}_{.2}$, ..., $\bar{X}_{.j}$ (i=1,2,3...,I; j=1,2,3...,J).

To test the null hypothesis (i.e. all the means $\bar{X}_j$ are equal) we compare the estimate of variance based on the variation among the $\bar{X}'s$ and estimate of

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variance based on the variation within samples [3-7].

The extent to which the sample mean can be expected to fluctuate, or vary, due to chance i.e. the uncertainty in the determination of the mean called estimated error of the mean is

$$S_{\bar{X}} = \frac{S_1}{\sqrt{J}} \tag{1}$$

Variance of the sample mean of observation

$$S^2_{\bar{X}} = \frac{S^2_1}{J} \tag{2}$$

i.e.

$$S^2_1 = J S^2_{\bar{X}} \tag{3}$$

The variance based on the variation within the sample can be estimated from column variance S^2_j, j=1,2,3...,J; by averaging them,
i.e.

$$S^2_2 = \frac{\Sigma^J_{j=1} S^2_j}{J} \tag{4}$$

Now we have two estimates of variance, one from equation (3) and other from (4). We calculate

$$F = \frac{S^2_1}{S^2_2} \tag{5}$$

The null hypothesis is rejected if F exceed F_α, the F–distribution to some α level of significance. F_α may be looked up in appropriate table. If we compare the means of k random samples of size n, we take $k-1$ degrees of freedom for numerator and $k(n-1)$ degrees of freedom for the denominator.

The amplitude of Forbush decrease is usually $\sim 5-10\%$ while variation during a solar cycle is $\sim 20-30\%$. Therefore, it is essential to correct (transfer) the data (remove the solar cycle effect) before subjecting the data to significance test for Forbush effect [1].

3. Results

The test results for three groups, without correction (Table 1), show that the decrease in cosmic ray intensity due to only III group (SC$\geq$ 40γ) shocks is statistically significant.

However, when the test results were obtained after the correction of data (for correction procedure, see [1]), the observed decrease due to II group (25 <SC< 40) and group III (SC$\geq$ 40γ) shocks is significant (see Table 1) and the observed decrease is not merely due to the random fluctuations of the sampling. However, the decrease due to group I ($SC \leq 25\gamma$) is insignificant. Standard errors of mean on each epoch has also been plotted in Fig. 1, before (left) and after (right) corrections.

Table 1. Calculated F values

Group	Data	F	Result	$F_{0.05}$
I(SC$\leq 25\gamma$)	Uncorrected	0.12	Insignificant	1.80
	Corrected	0.90	Insignificant	1.80
II($25\gamma < SC < 40\gamma$)	Uncorrected	0.64	Insignificant	1.85
	Corrected	6.95	Significant	1.80
III(SC$\geq 40\gamma$)	Uncorrected	3.05	Significant	1.62
	Corrected	13.85	Significant	1.62

4. Discussion

Sudden commencement (SC) of geomagnetic storms signifies the arrival of interplanetary shocks at earth [2]. It is known that all the shocks arriving at earth do not produce observable effect in cosmic ray intensity recorded at earth, by neutron monitors. Therefore, in addition to illustrate the test procedure (described in this paper), the results of Chree analysis investigate the effectiveness of shocks of different SC amplitude on transient modulation of cosmic rays.

5. Conclusions

A procedure is described to test the significance of average 'effect' observed in cosmic ray data. It is suggested that data, having other superposed effect of comparable amplitude, should be corrected for such effects before the test of significance for 'genuine' signal. Using this method it is demonstrated that the decrease observed in cosmic ray intensity following shocks with SC amplitude $\leq 25\gamma$ can not be treated as significant. However, the decrease due to shocks with higher SC amplitude $> 25\gamma$ may be treated as significant at 5% level.

6. Reference

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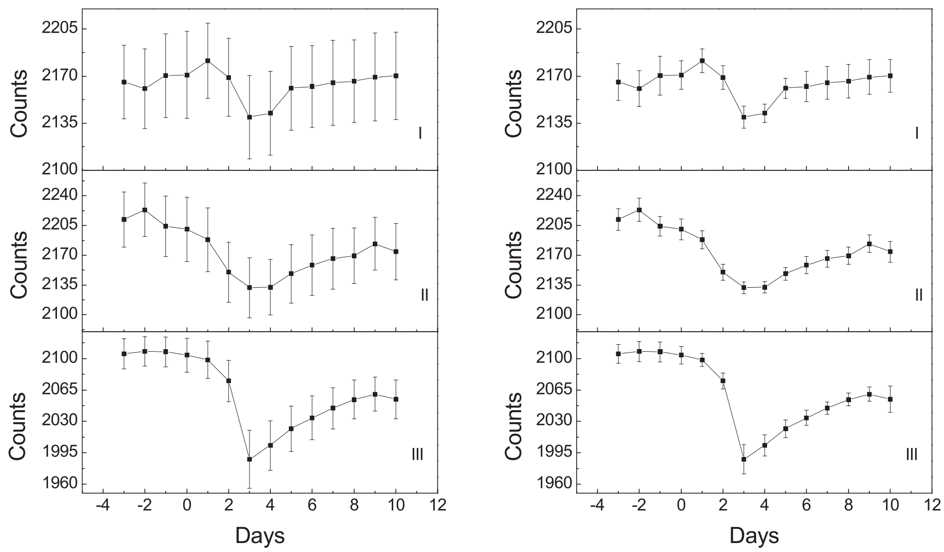

Fig. 1. Superposed epoch analysis results of cosmic ray intensity alongwith the standard error of mean bars before (left) and after (right) the correction, with respect to interplanetary shocks divided in three groups according to SC amplitude due to shocks; group I ($5\gamma < SC < 25\gamma$), group II ($25\gamma < SC < 40\gamma$), and group III ($SC \geq 40\gamma$).