Distribution Functions of Muons in Inclined Showers Registered by Auger Observatory

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Abstract

The Lateral Distribution Function (LDF) and Energy Distribution Function (EDF) of muons in Ultra High Energy (UHE) cosmic ray induced air showers are of great importance for testing interaction models at high energies, and also in the search for exotic particles, such as WIMPs and UHE neutrinos. Direct measurements of the muon distribution functions are extremely difficult to do, because of the need for shielded muon detectors as well as an extensive surface detector with fine binning. Inclined showers are easier to investigate from the point of view that on the ground level there are only muons and muon interaction products. Due to the large slant depth there is no remaining electromagnetic component. Therefore there is no need to separate the electromagnetic part of the signal from muon’s part, as is necessary for vertical air showers. In this paper we will present a possible semi-model independent technique for measuring distribution functions of muons in inclined showers and discuss some questions of sewing the LDFs of vertical and horizontal air showers.

1. Introduction

Auger surface detectors (SDs) are water cherenkov tanks of cylindrical shape (r=1.78 m and h=1.2 m) arranged in triangular cells and separated by 1.5 km. The important features for horizontal air shower (HAS) (zenith angles \( \Theta > 60^\circ \)) detection are: the effective detectional area of the SDs increase with increasing \( \Theta \); the signal from a single muon is proportional to the length of its track inside a SD and weakly depends on the muon’s energy in our range of interest.

Reconstruction of HAS using SD technique could be divided as follows:

1. From the timing of each SD, we get the shower axis direction.

2. Corrections for atmospheric parameters and the Earth’s magnetic field are incorporated.
3. Since the electromagnetic part is absorbed, we can fit muon density on the ground to our data as a function of primary’s energy $E_{\text{prim}}$, and the depth of first nuclear interaction $X_1$.

Details of last two steps and a possibility of some characteristic hadronic interaction function measurement, will be discussed in this paper.

2. Atmosphere

The geometry of muon generation in HAS is shown in Fig. 1. The shower zenith angle is $\Theta$. $L_s$ is a distance from the muon generation points (A and B) to the shower plane, that contains the point where the muon impacts (C and D). $X$ is the slant depth of the muon generation point along the shower axis.

Coordinates in the shower plane ($x'$ and $y'$ with distance from the shower axis $R' = \sqrt{x'^2 + y'^2}$) are chosen so that the $y'$-axis is along the Earth’s magnetic field component perpendicular to the shower axis ($B_\perp$). Coordinates on the ground are $x$ and $y$ with $R = \sqrt{x^2 + y^2}$. The center for coordinates is at the shower core. Components of muon momentum at the generation point are $p_\perp$ (perpendicular to the shower axis) and $p_\parallel$ (parallel to the shower axis).

Muon density profiles are defined by summing over muons created at different distances $L_s$ and angles $\beta$, but crossing the same region at the ground.

2.1. Influence of atmospheric density profiles on muon densities

As a primary particle enters the atmosphere, it interacts producing a hadronic shower. The most important channel of muon production is the decay of a pions. The number of muons generated at a depth $X$ depends on $E_{\text{prim}}$ and on the ratio $\frac{L_{\text{decay}}}{L_{\text{hadron}}}$ [3], where $L_{\text{decay}}$ is the characteristic length of pion decay and $L_{\text{hadron}}$ is the characteristic length of hadronic interaction at a given $X$.

The number density of generated muons along the shower axis is $G(X)$ and the total number of muons produced in a given interval of $X$ is: $N_{\mu} =$
\[ \int_{X_A}^{X_B} G(X) dX. \]

The muon angular spectral density function \( \Psi(p_{\perp}, p_{\parallel}, X) \) is the ratio of muons generated at \( X \) with \( p_{\perp} \) and \( p_{\parallel} \) to the total number of muons generated at \( X \).

The density of muons generated at \( L_s \) with momentum \( p \) at distance \( R' \) from shower axis can be written as:

\[
\rho(R', L_s, p) = \frac{1}{2\pi R'} \frac{L_s}{R'^2 + L_s^2} \frac{\partial X(L_s)}{\partial L_s} G(X(L_s)) \Psi\left( \frac{p \cdot R'}{\sqrt{L_s^2 + R'^2}}, \frac{p \cdot L_s}{\sqrt{L_s^2 + R'^2}}, X(L_s) \right) \cdot p.
\]

(1)

It’s worth noting, that the muon energy threshold and muon energy attenuation in the atmosphere could be naturally incorporated directly into the \( \Psi \) function.

The resulting muon number profiles on the ground can be calculated by projecting \( R'(x', y') \rightarrow R(x, y) \) at a given \( L_s \) and then integrating \( \rho(R', L_s, p) \) over \( L_s \) from 0 to the top of the atmosphere and over \( p \) from 0 to \( \infty \).

2.2. Parametrization of \( \Psi \) and \( G \) functions

The ARIES simulation program with a QGSJET hadronic interaction model was used to obtain the following parametrization.

\( G(X) \) shape was parametrized as:

\[
G(X) = \frac{1}{X_o} \cdot \left( \frac{X}{X_o} \right)^{\alpha - 1} \cdot \exp\left( -\frac{X}{X_o} \right),
\]

(2)

where \( X_o \) depends on \( E_{\text{prim}} \), and \( \alpha \) depends upon \( X_1 \) (i.e. \( \frac{L_{\text{decay}}}{L_{\text{hadron}}} \)). The function \( \Psi \) can be represented in the following way:

\[
\Psi(p_{\perp}, p_{\parallel}, X) = \frac{\Psi_\Delta(\log_{10}(p_{\perp}), \log_{10}(p_{\parallel}), X)}{p_{\perp} \cdot p_{\parallel}},
\]

(3)

and \( \Psi_\Delta \) can be parametrized as a Gaussian with the center at \( (p_{\perp}^{\text{max}}, p_{\parallel}^{\text{max}}) \) rotated by angle \( \alpha \). In the rotated coordinates \( \xi \) and \( \zeta \):

\[
\Psi_\Delta = N_{\text{norm}} \cdot e^{-\left( \frac{(\xi(p_{\perp} - p_{\perp}^{\text{max}}, p_{\parallel} - p_{\parallel}^{\text{max}}, \alpha))^2}{\eta} - \left( \frac{\zeta(p_{\perp} - p_{\perp}^{\text{max}}, p_{\parallel} - p_{\parallel}^{\text{max}}, \alpha))^2}{\gamma} \right)^2 \right)},
\]

(4)

\( p_{\perp}^{\text{max}} \) depends very weakly on \( X_1 \) and \( X \). It is mostly defined by the kinematics of the process, i.e. \( E_{\text{prim}} \). \( p_{\parallel}^{\text{max}} \) has a complicated dependence on \( E_{\text{prim}}, X_1 \) and \( X \). We assume that \( \eta \) and \( \gamma \) do not depend upon \( E_{\text{prim}} \) and \( X_1 \).

As the \( \frac{L_{\text{decay}}}{L_{\text{hadron}}} \) increases, fewer, but more energetic, muons are produced.
3. Earth’s Magnetic Field $B$

In HAS, the influence of $B$ becomes important, because of larger distances traveled. The perpendicular muon shift $\delta x'$ due to the perpendicular magnetic field $B_\perp$ while travelling a distance $L_s$ is estimated as [1]:

$$\delta x' = \frac{eB_\perp L_s^2}{2p},$$  \hspace{1cm} (5)

where $p$ is initial muon’s momentum. The muon profiles on the ground will be distorted, depending on the muon energy distribution. If a muon with momentum $p$ has coordinates $R' = \sqrt{x'^2 + y'^2}$ in the shower plane, coordinates will be $R'_{B_\perp}(p, L_s) = \sqrt{(x' \pm \delta x')^2 + y'^2}$ (+ or − depending on muon’s charge), when the magnetic field is turned on. The modified muon densities in the shower plane can be calculated as:

$$\rho_{B_\perp}(x', y') = \int_0^{+\infty} dL_s \int_0^{+\infty} dp \rho(R'_{B_\perp}(p, L_s), L_s, p),$$  \hspace{1cm} (6)

from which densities on the ground can be calculated (see Fig. 1).

In the range of angles $\Theta$ from 60° to 75°, multiple Coloumb scattering of the muons plays an important role [2], as well as the products of the muons interactions in the atmosphere. These are not yet taken into account in the current framework.

4. Conclusions

We have developed a framework for calculating muon densities on the ground for HAS, having four input parameters: $E_{prim}$, $X_1$, $\Theta$, $\Phi$. This framework could be used in order to reconstruct HAS in Auger experiment. This framework could also be used for probing the validity of our hadronic interaction model.

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References