Energetic Particle Mean Free Path in the Wave Heated Solar Wind

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Abstract

We present simple analytical expressions for the power spectrum of cascading Alfvén waves and the resulting solar energetic particle (SEP) mean free path in the solar wind. The model can reproduce the short coronal mean free path required for efficient acceleration of SEPs in coronal shock waves as well as a longer interplanetary mean free path required for a rapid propagation of the accelerated particles to 1 AU.

1. Introduction

Recent observations of high and anisotropic ion temperatures in the solar corona [4] give observational support to models employing the cyclotron heating mechanism to heat the plasma on open magnetic field lines. In these models, the energy input for heating the plasma comes from Alfvén waves created at the solar surface. The waves propagate until their frequency is comparable to the local ion-cyclotron frequency, and the wave energy is absorbed by the plasma ions via the cyclotron-resonance. Solar energetic particles (SEPs) interact strongly with the waves responsible for heating the corona [7]. The same waves that heat the solar corona can help to rapidly accelerate SEPs in coronal shock waves and, thus, explain particle acceleration in small SEP events, where self-generated waves can not explain the rapid acceleration. On the other hand, observed parameters of SEP transport in the solar wind give constraints to the wave-heating models, limiting the magnitude and spatial extent of wave heating in the solar wind.

2. The Wave Power Spectrum and the SEP Mean Free Path

We consider Alfvén waves propagating in the solar corona and solar wind (see, e.g., [3]). An equation governing the power spectrum $P(f, r)$ of outward-propagating Alfvén waves in the solar wind is, in steady state, given by [6]

$$\frac{v_\mathrm{A}}{AV}\frac{\partial}{\partial r}\left(\frac{AV^2}{v_\mathrm{A}}P\right) = -\frac{\partial F}{\partial f}, \quad F = 2\pi\alpha\alpha_1\frac{v_\mathrm{A}}{V}\frac{f^{5/2}P^{3/2}}{B}. \tag{1}$$

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Here f is the wave frequency, $V = u + v_{\mathrm{A}}$, and u and v_{A} the solar-wind and the Alfvén speed, both functions of the heliocentric distance r. The given form of the spectral flux function, $F(f,r)$, corresponds to the Kolmogorov cascade phenomenology. It is proportional to the cascade constant $\alpha = 1.25$ and to the square root of the ratio of the inward and outward wave intensity, α_1, which is a model parameter taken to depend on r, only. The flux-tube cross-sectional area A is inversely proportional to the magnetic field B, which is taken to point at the radial direction.

The Alfvén-wave power spectrum can be solved in an analytic form, if α_1 is a given function of position. In this case, the spectrum can be given as [8]

$$P = \frac{v_{\mathrm{A}}}{AV^2}\frac{A_\odot V_\odot^2}{v_{\mathrm{A}\odot}}\frac{f_0^{5/3} I\big(x(f),\tau(r)\big)}{f^{5/3}} P(f_0, r_\odot), \tag{2}$$

where the dimensionless function $I(x,\tau)$ fulfills the equation

$$I(x,\tau) = g\Big(x + I^{1/2}(x,\tau)\tau\Big), \text{ with } g = I(x,0) = \frac{P(f_0 x^{-3/2}, r_\odot)}{x^{5/2}\, P(f_0, r_\odot)}, \tag{3}$$

$$x = \left(\frac{f_0}{f}\right)^{2/3}, \text{ and } \tau = 2\pi f_0 \epsilon_P^{1/2}\alpha \int_{r_\odot}^{r} \frac{\alpha_1(r')V_\odot v_{\mathrm{A}}(r')}{V^3(r')}\left(\frac{n_{\mathrm{e}\odot}}{n_{\mathrm{e}}(r')}\right)^{1/4} \mathrm{d}r'. \tag{4}$$

Here, f_0 is an arbitrary normalization frequency, n_{e} is the electron density, and $\epsilon_P \equiv f_0 P(f_0, r_\odot)/B_\odot^2 \ll 1$ is a dimensionless constant. All quantities indexed with $\odot$ refer to the values at the solar surface. When deriving the spectrum, conservation of mass and quasi-neutrality in an electron–proton plasma are used, i.e., $An_{\mathrm{e}}u = \text{const}$.

In the special case of $P(f, r_\odot) = \epsilon_P B_\odot^2/f$, the initial dimensionless spectrum becomes $g(x) = x$. We will restrict the discussion to this special case in this paper. In this case, the power spectrum of the Alfvén waves can approximated by

$$P(f,r) = \frac{v_{\mathrm{A}}}{AV^2}\frac{A_\odot V_\odot^2}{v_{\mathrm{A}\odot}}\frac{\epsilon_P B_\odot^2}{f\left[1 + (f/f_{\mathrm{c}})^{2/3}\right]}, \text{ with } \frac{1}{f_{\mathrm{c}}(r)} = \frac{\tau(r)}{f_0}. \tag{5}$$

The spectrum is, thus, a broken power law with a spectral index of -1 and $-5/3$ below and above the spectral break point frequency $f_{\mathrm{c}}(r)$, which decreases with heliocentric distance. Note that such form of the power spectrum is supported by observations in the solar wind [2].

In the case of a wave-heated solar wind, the power spectrum of the Alfvén waves determines the SEP mean free path (excluding electrons). Taking the Alfvén waves to be linearly polarized quasi-parallel propagating waves, the SEP mean free path $\lambda(v,r)$ can be given as the standard integral of the inverse pitch-angle diffusion coefficient over pitch-angle cosine μ,

$$\lambda = \frac{3v}{8}\int_{-1}^{+1} \mathrm{d}\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}}; \quad D_{\mu\mu} = \frac{\pi}{4}\,\Omega\,(1-\mu^2)\frac{f_{\mathrm{r}} P(f_{\mathrm{r}}, r)}{B^2}; \quad f_{\mathrm{r}} = \frac{\Omega}{2\pi}\frac{V}{v|\mu|}. \tag{6}$$

Here, Ω and $f_{\rm r}$ are the (angular) particle cyclotron and resonant wave frequency, respectively. Substituting the form of the power spectrum gives

$$\lambda = \frac{2}{\pi\epsilon_P}\frac{v}{\Omega}\left[1 + \frac{27}{7}\left(\frac{\Omega V}{2\pi f_{\rm c} v}\right)^{2/3}\right]\left(\frac{n_{\rm e}}{n_{\rm e\odot}}\right)^{1/2}\frac{V^2}{V_\odot^2}. \tag{7}$$

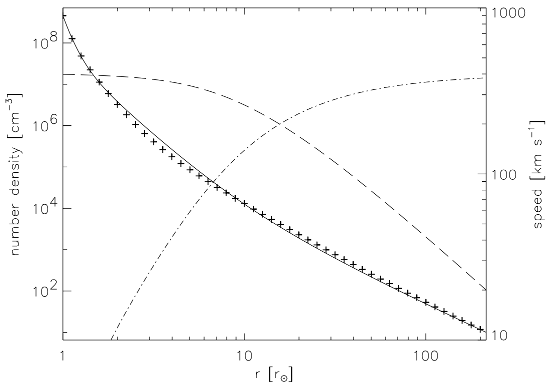
When performing the integral, we have assumed the broken power-law spectrum of the waves to continue to infinite frequencies. In reality, the spectrum has a cut-off at the dissipation frequency $f_{\rm H}$. In an electron–proton plasma, this frequency is close to the proton-cyclotron frequency of the plasma, $f_{\rm H} = \alpha_f \Omega_{\rm p}/2\pi$, where the value of the parameter α_f depends on the plasma parameters [5,7], although it is frequently taken to be a constant (e.g., [3]). Thus, the model for the mean free path can be applied only at speeds $v \gg \Omega V/(2\pi f_{\rm H})$.

We have used an ad-hoc model of the solar wind to calculate the SEP mean free path in the inner heliosphere ($r < 1$ AU). Assuming a constant value of $V = 400$ km s^{-1}, the magnetic field $B = 1.3\,(r_\odot/r)^2[1 + 1.9\,(r_\odot/r)^6]$ G, and the electron density of 10 cm^{-3} at 1 AU gives the density, flow speed, and Alfvén speed profiles depicted in Fig. 1. In Fig. 2, the resulting 10-MeV proton mean free path is plotted for wave parameters $\epsilon_P = 5\,10^{-5}$ and α_1 that has a constant value at $r > 10\,r_\odot$ and increases linearly from 0 to this value at $r_\odot < r < 10\,r_\odot$. Such values of ϵ_P are needed to produce a solar wind fulfilling observational criteria of mass flux and speed [5]. The values for α_1 are taken from papers modeling the solar wind expansion [3,5]. The (solid) curve in Fig. 2, representing cascading Alfvén waves, connects a very short mean free path close to the solar surface ($r < 2\,r_\odot$) with a larger, spatially almost constant value at larger distances from the Sun. Thus, the model may offer a consistent explanation of both efficient SEP acceleration at coronal shocks (requiring small λ) in small SEP events, where efficient generation of the Alfvén waves by the energetic protons themselves is not possible, and of the subsequent rapid interplanetary propagation from the acceleration site to the observer.

Self-consistent modeling [5] of the Alfvén-wave propagation and the solar wind expansion in case of no cascade term in the wave transport equation ($\alpha_1 = 0$) produces too small SEP mean free paths in the solar wind. We have demonstrated that cascading can dramatically increase the values of the mean free path in the solar wind. Self-consistent modeling of the wind expansion with cascading waves is in progress.

3. List of Symbols

r	heliocentric distance, cm	$r_\odot$	solar radius $= 6.96\,10^{10}$ cm
u	solar wind speed, cm s^{-1}	$v_{\rm A}$	Alfven speed, cm s^{-1}
B	magnetic field, G	$n_{\rm e}$	electron number density, cm^{-3}



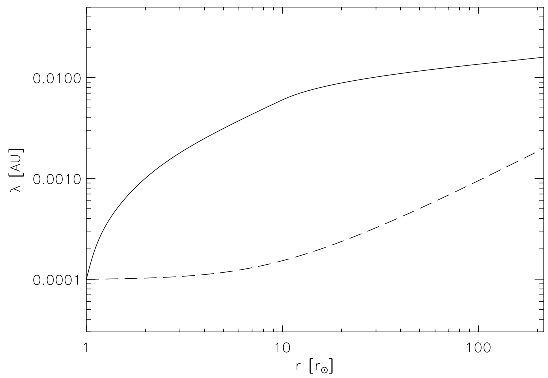


Fig. 1. Solar wind electron density (solid), Alfvén speed (dashed), and flow speed (dot-dashed). Semi-empirical electron densities (cross) are also given [1].

Fig. 2. 10-MeV proton mean free path for the solar-wind model depicted in Fig. 1. Results for $\alpha_1 = 0.05$ (solid) and $\alpha_1 = 0$ (dashed) at $r > 10\,r_\odot$ are shown.

A	flux-tube cross section, cm^{-2}	Ω_p	proton-cyclotron frequency, s^{-1}
P	wave power spectrum, G^2 Hz^{-1}	F	spectral flux function, G^2 s^{-1}
ϵ_P	wave spectrum parameter	V	inertial-frame wave speed, cm s^{-1}
α	cascade constant = 1.25	α_1^2	inward-outward wave ratio
I	dimensionless power spectrum	x	dimensionless f-variable, Eq. (4)
τ	dimensionless r-variable, Eq. (4)	f	inertial-frame wave frequency, Hz
f_0	normalization frequency, Hz	f_c	spectral break-point frequency, Hz
f_r	resonant wave frequency, Hz	f_H	dissipation frequency, Hz
α_f	$= 2\pi f_\mathrm{H}/\Omega_\mathrm{p}$	v	particle speed, cm s^{-1}
μ	particle pitch-angle cosine	Ω	particle cyclotron frequency, s^{-1}
$D_{\mu\mu}$	pitch-angle diffusion coefficient, s^{-1}	λ	scattering mean free path, cm

Quantities indexed with $\odot$ refer to values at the solar surface, e.g., $A_\odot = A(r_\odot)$.

4. References

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