Conditional Statistics of Magnetic Turbulence and the Lateral Transport of Solar Energetic Particles

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Abstract

The transport of energetic particles perpendicular to the mean magnetic field in space plasmas long has been viewed as a diffusive process. However, there is an apparent conflict between recent observations of solar energetic particles (SEP): 1) SEP from impulsive solar flares can exhibit “dropouts” in which the intensity near Earth repeatedly disappears and reappears. This indicates that the distribution of SEP in space is highly filamentary, with very little lateral diffusion across these boundaries. 2) Observations by the IMP-8 and Ulysses spacecraft, while they were on opposite sides of the Sun, showed similar time-intensity profiles for many SEP events. This indicates that particles often undergo rapid lateral diffusion. We explain these seemingly contradictory observations using a theoretical model, supported by computer simulations, in which many particles are temporarily trapped within topological structures in statistically homogeneous turbulence, and ultimately escape to diffuse at a much faster rate.

1. Introduction

While spatial inhomogeneities in SEP distributions have been known for decades, they were originally attributed to large-scale disruption of magnetic connection with the Sun, e.g., magnetic sector boundaries, fast/slow solar wind boundaries, large-scale flux tubes or magnetic clouds, etc. However, the recent dropouts observed by the Advanced Composition Explorer (ACE) spacecraft for a large number of impulsive solar events [5] occur repeatedly and over such small scales (∼ 0.03 AU) that they cannot be attributed to large-scale features, and instead must be related to the spatial structure of the interplanetary magnetic field [2]. Indeed, we argue that dropouts are a signature of the topology of magnetic turbulence in the solar wind. SEP from impulsive solar events serve as a good probe of lateral transport because they arise from a localized source. Particle transport perpendicular to the mean magnetic field vs. time is generally attributed to the random walk of turbulent field lines vs. distance along the mean field. However, perpendicular transport of a diffusive nature cannot explain both the dropouts and the IMP-8/Ulysses observations; the latter imply such rapid diffusion that the small-scale dropouts would be washed out. It has been proposed
that fluid motions at the solar surface cause filamentary structures consistent with dropouts, but one must “switch off” the field line random walk when explaining the dropouts, and switch it on in order to explain large-scale lateral transport.

2. Turbulence Model and Analysis of Long-Time Diffusion

We propose to reconcile these observations in terms of a two-component model of solar wind turbulence that provides a useful explanation of both its magnetic statistics and the parallel transport of SEP. This assumes a constant (or slowly varying) mean magnetic field plus two components of transverse fluctuations. The “slab” component of turbulence \( \vec{b}_{\text{slab}} \) depends only on \( z \), the coordinate along the mean field, while the “2D” component \( \vec{b}_{\text{2D}} \) depends only on the perpendicular coordinates, \( x \) and \( y \). For 2D turbulence alone, magnetic field lines can remain trapped near certain \((x, y)\) coordinates because they always follow contours of a so-called potential function \( a(x, y) \) \[ \vec{b}_{\text{2D}}(x, y) = \nabla \times [a(x, y) \hat{z}] \], where \( a \hat{z} \) is the vector potential. Figure 1 shows a contour plot of \( a(x, y) \) for a specific representation of 2D turbulence with desired statistical properties (appropriate for the solar wind). The “◦” symbols in Figure 1 indicate O-points [local maxima or minima in \( a(x, y) \)] where the contours remain trapped within “islands” of the 2D turbulence (or filaments in three-dimensional space). We also indicate X-points, i.e., saddle points of \( a(x, y) \). Thus even turbulence with homogeneous statistical properties can have a topological structure.

The ensemble average statistics of the field line random walk were calculated by [4]. A diffusion coefficient, \( D \), is defined by \( \langle \Delta x^2 \rangle = 2D\Delta z \). Each turbulence component is associated with a value of \( D \); the overall value is \( D = D_{\text{slab}}/2 + \sqrt{(D_{\text{slab}}/2)^2 + (D_{\text{2D}})^2} \). Under normal solar wind conditions, \( D_{\text{slab}} \) is very small \( \approx 5 \times 10^4 \) AU. As an aside, the total diffusion coefficient can be estimated from the IMP-8 and Ulysses SEP data sets [3,6]. For most solar events shown by [6], the time-intensity profiles at the two spacecraft are very similar, in shape as well as absolute magnitude, immediately after the peak in particle intensity. Only Event 6 shows a distinctly diffusive rise at Ulysses before matching IMP-8 data in the decay phase. Therefore we have fit this most diffusive event, using a Reid profile [9] centered at the Archimedean field line of the flare site at the radial distance of Ulysses (2.35 AU), to provide a lower bound on the particle diffusion coefficient \( \kappa_\perp \) and (using the field line random walk concept) a conservative lower limit on \( D \). We obtain \( \kappa_\perp \geq 1.3 \times 10^{21} \text{ cm}^2 \text{ s}^{-1} \) and \( D \geq 0.02 \) AU, which are lower but of the same order of magnitude as previous estimates, e.g., [7].

In sum, we interpret that the IMP-8 and Ulysses observations require a total \( D > 0.02 \) AU, so the 2D random walk dominates the slab random walk, and \( \sqrt{\langle \Delta x^2 \rangle} \) is > 0.2 AU at Earth orbit. However, such ensemble average statistics cannot apply to observations of dropouts, because the dropouts correspond to filamentation over \( \sim 0.03 \) AU, which would be completely washed out.
3. Temporary Trapping by Small-Scale Turbulence

Instead of ensemble average statistics, let us now consider conditional statistics, depending on the initial location of a magnetic field line. If a field line is near an O-point, within an island of the 2D turbulence (see Figure 1), the 2D contribution to the random walk is suppressed. The field line is temporarily trapped, with diffusion at the much slower rate characteristic of slab turbulence. On the other hand, magnetic field lines that start outside islands are rapidly carried far away by the 2D turbulence.

In particular, suppose that particles are injected in a spatially localized region, say a circle of radius \( \rho \). Then \( z_1 = \rho^2/(4D) \) is a characteristic distance over which field lines outside islands diffuse out of the circle. Next, consider islands of diameter \( d \); \( z_2 = d^2/(16D_{\text{slab}}) \) is the typical distance scale over which field lines escape from an island, given diffusion due to the slab component. If slab diffusion is weak, we can have \( z_1 < z_{\text{obs}} < z_2 \), where \( z_{\text{obs}} \) is the distance of the observer. We suggest that dropouts are observed under these conditions. (Note that we identify dropouts with topological structures that develop in solar wind turbulence, not with initial motions at the solar surface.) Magnetic field lines (and the particles orbiting them) that start deep within islands mostly remain trapped, while those outside the islands have already escaped from the injection region, leaving gaps with a low density of particles. After a long distance the field lines spread rapidly, corresponding to rapid lateral diffusion of particles.

This idea is confirmed by computer simulations that trace field line trajectories in representations of 2D+slab turbulence for typical solar wind values, using Cartesian geometry for simplicity. In Figure 2, the upper left panel shows random initial locations within a circle, corresponding to the injection region where field lines are populated with SEP. Subsequent panels, cross-sections at longer distances along the mean field, show filamentary structures in the distribution of SEP. A spacecraft near Earth (\( z \approx 1 \text{ AU} \)) samples a transect through this highly inhomogeneous distribution. The simulation results are consistent with observed dropouts of 0.03 AU. At longer distances, essentially all field lines (and particles) have diffused away, leading to the rapid propagation of particles throughout the inner heliosphere at later times.

4. Discussion

For a wide injection region, \( z_1 > z_{\text{obs}} \) and dropouts should not be seen. Indeed, gradual flare/CME events inject particles over a much wider region and do not exhibit dropouts (see the similar argument of [2]). We confirm that our model explains this by a “control run” in which field lines are randomly distributed throughout simulation space. The distribution indeed remains uniformly random (which in the context of our model is required by Liouville’s theorem).

This new view of the perpendicular transport of energetic particles in space
plasmas can also reconcile another pair of apparently conflicting observations. Impulsive solar events selected for a strong SEP electron increase were shown to have a narrow distribution in solar longitude \cite{Reames1990}. This indicates only limited lateral spreading for the bulk of SEP, which we attribute to trapping within small-scale topological islands, representing a “core” region of high particle density. On the other hand, recent spacecraft observations of Type III radio bursts and associated SEP indicate that SEP electrons and ions can undergo broad lateral motion (up to $\sim 90^\circ$ in solar longitude) during their transport from the Sun to Earth orbit \cite{Cane2003}. In our view, this laterally extended but less intense “halo” of SEP corresponds to particles on field lines initially located outside local islands of 2D turbulence. Indeed, the absence of these halo SEP from the core region is manifest as dropouts.

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5. References
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