A wavelet-based approach to UHECR arrival direction analysis

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Abstract

A Mexican Hat wavelet is used to analyze the arrival directions of ultra-high energy cosmic rays (UHECRs) detected with the Yakutsk array. The purpose is a search for possible deviation from expected isotropic distribution in right ascension and on the celestial sphere. Necessary limitations are discussed in order to adapt an unbounded wavelet to a circle and sphere. As a result, significant anisotropy is found in the energy range between 10^{19} and 3.16×10^{19} eV: the chance probability for the isotropic distribution to have the wavelet amplitude exceeding observed one is less than 0.5%. The wavelet center is (2.3±1.3) h in right ascension, $52.5^0 \pm 7.5^0$ in declination; $10^0 < R < 20^0$ are the scale parameter limits. The result relevance is stressed by coincident evidences of anisotropy in the same energy range drawn from the Yakutsk array data using different methods of analysis - wavelet, harmonic and galactic latitude distribution.

1. Introduction

Cosmic rays in ultra-high energy domain $E > 10^{18}$ eV exhibit seemingly isotropic arrival direction distribution. However, there are several indications of anisotropy claimed by AGASA [3], Durham [8], Yakutsk [5] and other groups. In this paper we are dealing with wavelet analysis which has been already demonstrated to be going on well in many applications in various scientific fields [1,2]. The continuous isotropic wavelet transform of the function $f(x)$ is defined as

$$wv(R,\vec{b}) = \int d\vec{x} f(\vec{x})\Psi(R,\vec{b};\vec{x}); \quad \Psi(R,\vec{b};\vec{x}) = \frac{1}{R}\psi(\frac{|\,\vec{x}-\vec{b}\,|}{R}), \qquad (1)$$

where $wv(R,\vec{b})$ is the wavelet coefficient associated with the scale R at the point with coordinate $\vec{b}$. $\psi(|\,\vec{x}\,|)$ is the "mother" wavelet that is assumed to be isotropic.

The necessary and sufficient conditions for applicability of the wavelet transformation and synthesis are: i) compensation, $\int d\vec{x}\psi = 0$; ii) normalization, $\int d\vec{x}\psi^2 = 1$; iii) admissibility, $(2\pi^2)\int dq q^{-1}\psi^2(q) < \infty$, where $\psi(q)$ is the Fourier transform of $\psi(x)$.

pp. 341–344

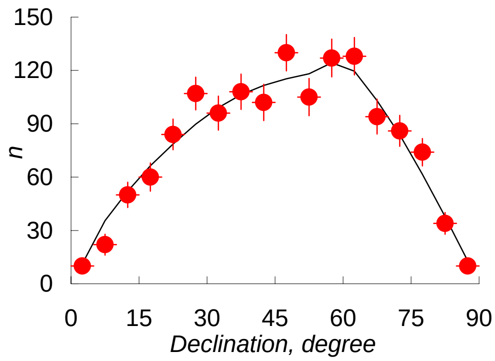

Fig. 1. A declination distribution of observed showers (points) above 5×10^{18} eV in comparison with expected isotropic spread (solid line). Statistical errors are shown by the vertical bars, horizontal ones indicate declination bins.

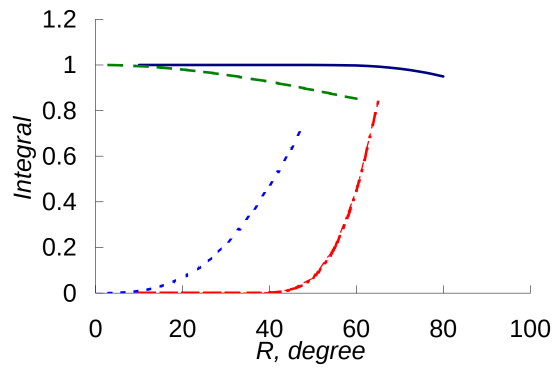

Fig. 2. Compensation and normalization integrals of the Marr wavelet on a 360^0 circle and on the sphere as a function of R. Captions: normalization curves are shown by solid (one-dimensional case) and dash (two-dimensional) lines; compensation curves are dash-and-dot line (1D) and dotted one (2D case).

2. A sample of the Yakutsk array data used in analysis

In the analysis, the data sample of 28726 showers are used above 10^{18} eV selected with zenith angles $\theta < 60^0$, axes inside the Yakutsk array perimeter. With this threshold energy and selection criteria the non-uniformity in the sky coverage in right ascension due to seasonal and diurnal variations of the array acceptance area is shown to be less than statistical uncertainties [7]. On the contrary, the exposure of any ground based array is a complicated function of declination which is formed by zenith angle dependence of the array aperture and exposition time of a celestial region in the diurnal cycle. Expected-for-isotropy distribution of showers in declination is shown (Fig. 1; derived converting $sin(2\theta)$ from horizontal system) for the data selection criteria in use, together with actual data of the Yakutsk array in the energy range where the attenuation of the particle density doesn't affect the picture.

3. One-dimensional Marr wavelet on the right ascension circle

The most appropriate isotropic wavelet to use in one-dimensional case is the Marr, or 'Mexican Hat' wavelet [6] given by $\psi(x) = \frac{2}{\sqrt{3\sqrt{\pi}}}(1 - x^2)exp(-x^2/2)$, where x is the distance from the wavelet center.

In order to apply the infinite Marr wavelet to the right ascension circle, we must restrict the scale parameter R within the range where all applicability conditions are valid. Fig. 2 shows that $R < 50^0$ condition is sufficient to apply one-dimensional Marr wavelet on the right ascension circle with an accuracy better

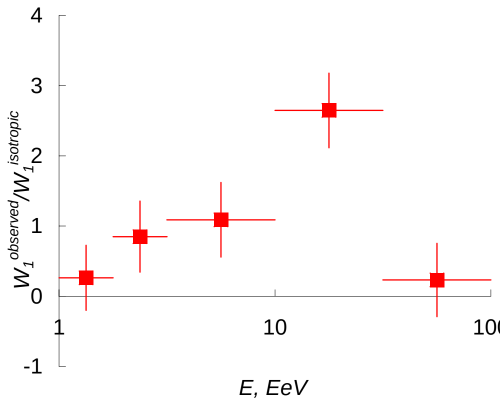

Fig. 3. One-dimensional wavelet amplitude in right ascension as a function of energy. Vertical bars are statistical errors for N random points; horizontal ones are energy bins. $5^0 < R < 50^0$.

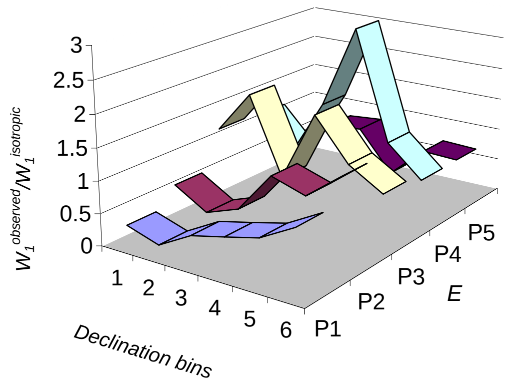

Fig. 4. Two-dimensional wavelet amplitude as a function of energy and declination. $R = 20^0$. Energy intervals are the same as in Fig. 3, while declination bins are $15^0 \times (1, ..., 6)$.

than 10%.

Although the continuous wavelet coefficient is zero in the case $f = const$, in the real world we have an artefact non-zero value due to the limited number of events N. To avoid this, one can use the Monte Carlo simulated isotropic wavelet coefficient for the observed number of showers in a particular energy bin as the expected value instead of zero.

The random variation of $wv(R, b)$ for fixed R (when $R = 50^0$, for instance) can be characterized by the first harmonic amplitude in b which tends to zero with rising N. So we have calculated wavelet first harmonic amplitude, W_1, of observed distribution in energy bins for given R in comparison with expected-for-isotropy value for N random points on the right ascension circle. Resulting observed/expected amplitude ratio is given (Fig. 3) of showers detected with the Yakutsk array.

There is energy bin $10^{19} < E < 3.16 \times 10^{19}$ eV where the observed amplitude $W_1^{observed}$ is sufficiently greater than the isotropic expectation: a chance probability for the uniform spread of 312 fake showers to have the amplitude greater than measured one is 0.5%. A wavelet maximum is at $\alpha = (2.3 \pm 1.3)h$.

4. Two-dimensional wavelet on the equatorial sphere

The isotropic Mexican Hat mother wavelet in the two-dimensional case has the form [6]: $\psi(\vec{x}) = \frac{1}{\sqrt{2\pi R^2}}(2 - \vec{x}^2/R^2)exp(-0.5\vec{x}^2/R^2)$. In order to adapt it on the equatorial sphere one has to measure the distance from the wavelet center, $\vec{x}$, as the spherical arc, and to restrict the scale parameter, R, so that applicability conditions are valid. It was shown (Fig. 2) that two conditions in the 2-D case restrict the parameter within the range $0^0 < R < 20^0$, to better than 10%

accuracy.

The expected declination spread of isotropic arrival directions is sufficiently non-uniform (Fig. 1). So we have calculated the wavelet transform as the sum of delta functions in both cases: for observed and expected isotropic distribution of N equatorial angles in each energy bin. Expected one is averaged over a sample of 1000 trials. Resulting observed/expected ratio is given (Fig. 4) vs. E, δ. Again, there is an energy bin, P4: $10^{19} < E < 3.16 \times 10^{19}$ eV, where the observed amplitude is significantly greater than isotropic one: $W_1^{observed}/W_1^{isotropic} = 2.83 \pm 0.51; \alpha_{max} = (2.3 \pm 1.3)h; \delta_{max} = 52.5^0 \pm 7.5^0; 10^0 < R < 20^0$.

5. Conclusions and discussion

As it was shown earlier [5], the first harmonic amplitude of the Yakutsk array data distribution in right ascension sufficiently deviates from the expected-for-isotropy value in the vicinity of $E = 10^{19}$ eV. Another indication was found analyzing galactic latitude distribution of these data: a significant north-south asymmetry [4] in the same energy region. Our finding with wavelet analysis confirms these hints, and is further strengthening the evidence of the anisotropy in arrival directions of UHECRs detected with the Yakutsk array in the interval $10^{19} < E < 3.16 \times 10^{19}$ eV.

Due to space localization of the Marr wavelet we can point out a sky area $\alpha = (2.3 \pm 1.3)h,\ \delta = 52.5^0 \pm 7.5^0$ where an excess UHECR flux is detected in our data. The scale parameter is limited within $10^0 < R < 20^0$.

The anisotropy in arrival directions revealed in the Yakutsk array data by three different methods of data handling can be attributed to non-zero galactic nuclei fraction [4]. These nuclei of average $Z \sim 10$ can amount up to 10% of the primary isotropic flux at energy $E = 10^{19}$ eV. In this case the hypothesis is congruent to other indications of the excess flux of nucleons from galactic plane/center at $E \sim 10^{18}$ eV (for example: [3]).

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