Advective Diffusion Propagation Model for Galactic Cosmic Rays above 10^{12} eV

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Abstract

Based on the detailed research for the diffusive motions of protons and nuclei with energies more than 10^{12} eV in the galactic turbulent magnetic fields, we calculated the residence times of cosmic rays in the galactic disk with solving an one dimensional advective–diffusion equation. Consequently, this simple model predicts the observed features including energy dependences of the flux for the all–particle and the components, the knee in the energy spectrum, the chemical composition and the anisotropy, with assuming the parameters for the equations such as the magnetic field strength and the galactic wind velocity.

1. Diffusive motion of charged particles in turbulent magnetic fields

The magnetic field in the Galaxy is nearly parallel to the spiral arms and in addition to this regular component, the irregularities of roughly same strength exist. The scale length of the irregularities(L_{irr}) is estimated $10 - 100$pc with large uncertainty. We examined the equation o f motions of 1000 test particles started at an origin in the turbulent magnetic field . We assumed that the regular and irregular components of the galactic magnetic field have a same order of magnitude of their energy density, as suggested by many observations of the rotation measure, and assumed that the energy density spectrum has a Kolmogorov spectrum.

By the calculated position $\Delta\mathbf{x}$ and the time after Δt , we obtained the Fokker–Planck coefficients along and perpendicular to the regular magnetic field lines. With our calculations, the Fokker–Planck coefficients at the time more than several thousands years after an injection are constant.

The relation between the Fokker–Planck coefficients($D_\perp$ and $D_\parallel$) and the particle energy or normalized Larmor radius r_g/L_{irr} could be expressed by[2]

$$\frac{D_{ii}}{cL_{irr}} = \left(\frac{D_{ii}}{cL_{irr}}\right)_0 \left(\frac{r_g}{L_{irr}}\right)^\alpha \tag{1}$$

The fitting parameters of equation (1) for the calculated diffusion coefficients are listed on Table 1..

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Table 1. The fitting parameters of equation (1).

parameter	parallel	perpendicular
α	0.6018 ± 0.0218	0.6627 ± 0.0142
$\log(D_{ii}/CL_{ii})_0$	0.1893 ± 0.0281	-1.481 ± 0.018

2. Advective–diffusion of cosmic ray particles

For protons with energies 10^{14}eV, $D_\perp$ is 0.005 pc^2/year for $3\mu G$ and $L_{irr} =$ 100 pc. With this value, we could estimate the residence time (τ_R) in the galactic disk with the half thickness $l \sim 150$ pc, $\tau_R = l^2/D_\perp \sim 5 \times 10^6$ years. On the other hand, an extrapolation of the leaky box model to 10^{12} eV leads a residence time of 2×10^5 years. In a different way, the observed anisotropy amplitude predicts the diffusion velocity $v_D \simeq 300$km/s, and the averaged residence time is estimated as $l/v_D \simeq 5 \times 10^5$ years. So that, an idea of the diffusive leakage of cosmic ray particles perpendicular to the galactic magnetic field apparently contradicts to the observations.

The magnetic field lines of the loop and filament structures are open and perpendicular to the galactic disk. In these structures of the disk the confinement of cosmic rays is not strong and the particles are leaking more rapidly. Here we assume that the cosmic ray particles are leaking at these structure with diffusive motions along with the magnetic field lines and also outflows with galactic wind of velocity v_g, *i.e.*, advective–diffusion processes with $D_\parallel$ and v_g. The leakage of cosmic rays is expressed with the following one dimensional advective–diffusion equation,

$$\frac{\partial n}{\partial t} + v_g \frac{\partial n}{\partial x} = \frac{\partial}{\partial x}\left(D_\parallel \frac{\partial n}{\partial x}\right) + Q \tag{2}$$

With the numerical calculations of the advective–diffusion equation with position dependent magnetic field strength, we estimated the residence time τ_R which is defined as a time for the number of cosmic ray particles in a confinement volume to reach $1/e$ of a initial value, for cosmic ray particles with energies $10^{12} - 10^{17}$eV for various pairs of the parameters L_{irr} and v_g. In these calculations we took the confinement volume length of 300 pc of which a cosmic ray source $Q = Q_0\delta(t)$ is located at the middle. Here we assumed that the strength of the galactic magnetic field is $3\mu G$ for regular and irregular components, and decreases exponentially with a scale height of 1 kpc.

The calculated residence times for cosmic ray protons for different advection velocities are shown in Fig. 1(a). Also, the residence times for different components of cosmic rays are shown in Fig. 1(b). Apparently, the energy dependence of τ_R is power law ($\propto E^{-\gamma}$) and the indices vary at $\sim 10^{14}$eV. The leakage for lower energy cosmic rays is dominated by advection because τ_R apparently depends on v_g. In contrast, for higher energy, the leakage of cosmic rays is dominated by diffusion, and τ_R has same power law energy dependence as the diffusion coefficient in equation (1). Moreover, the energies of bending points on the curves of τ_R in Fig. 1(b) obviously depend on charge of particles, and τ_R are smaller for lighter nuclei, so that for higher energy, the averaged mass number of cosmic ray particles is expected to increase with energy. Furthermore, the anisotropy amplitude is expected to be inversely proportional to τ_R, so that the calculated τ_R of Fig. 1(b) predicts that the anisotropy amplitude have the tendency to be almost constant up to $\sim 10^{14}$eV , and above this energy, increase with energy. This feature and the estimated values are consistent with the measured amplitudes[3].

3. Discussions and Conclusions

Using the calculated residence times τ_R, the energy spectra for each components, the all particle spectrum and the chemical composition are estimated with the simple assumed relation $N(E) = Q(E)\tau_R(E)$. The relative abundances for each component is estimated from the direct measurements of SOKOL[4] and CRN[5] and the spectral indices of $Q_i(E)$ are taken to fit the spectra of RUNJOB[1] collaboration, except the Fe spectrum, so that γ=2.7 for protons and He, 2.6 for CNO and Ne–Si, and 2.4 for Fe, respectively. The calculated all–particle spectrum is normalized with the direct measurement at 10^{12} eV. The other parameters in this calculation are assumed $v_g = 490$ km/s and $L_{irr} = 50$ pc. These values agree with the observations of the galactic magnetic fields and the predicted model of the galactic wind. The all–particle spectrum of the model is shown in Fig. 2(a). This curve is not inconsistent with the measurements, and has the spectral index jump of -2.7 to -3.0 over the knee. This steepening reflects the transition of dominant processes from outflow to diffusion. Furthermore, the calculated curves of $\langle \ln A \rangle$ as shown in Fig. 2(b) shows good agreement with the satellite, the balloon– borne measurements, and BASJE–MAS[6] and some observations up to 10^{17}eV.

References

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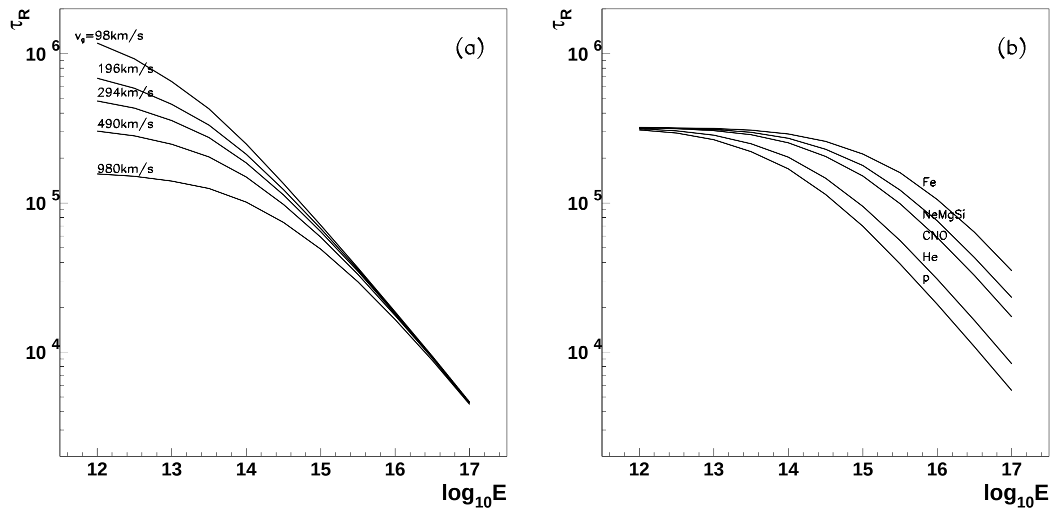

Fig. 1. (a)The residence time τ_R predicted from the advective diffusion model for cosmic ray protons. This figure shows τ_R for different advection velocity, *i.e.*, the galactic wind velocity, v_g. For all the calculations in this figure L_{irr} is fixed 100 pc. (b)The residence time τ_R for different primary components of cosmic ray particles. For all the calculations in this figure v_g is fixed 5×10^{-4} pc/years = 490km/s and L_{irr} is fixed 50 pc.

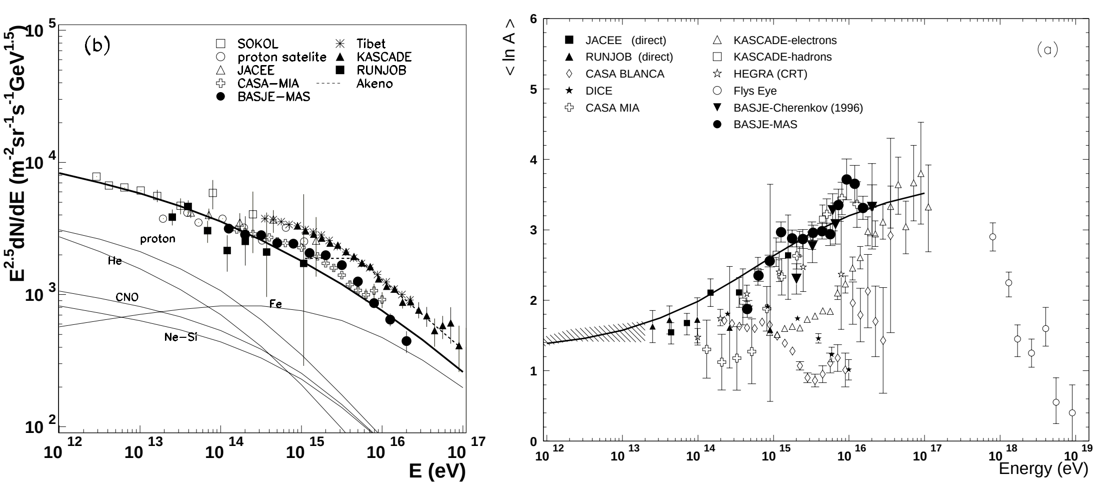

Fig. 2. (a)Calculated all particle spectra with the advective–diffusion model. (b)The averaged mass number, $\langle \ln A \rangle$, calculated with the model. The measured values with various different types of observations are also plotted on this figure. For the references of these measurements, please see the paper by S. Ogio et al.[6].