Wave Effects in Gravitational Lensing of Gravitational Waves from Chirping Binaries

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Abstract

In the gravitational lensing of gravitational waves, the wave optics should be used instead of the geometrical optics when the wavelength λ of the gravitational waves is longer than the Schwarzschild radius of the lens mass M_L. For the gravitational lensing of the chirp signals from the coalescence of the super massive black holes at the redshift $z_S \sim 1$ relevant to LISA, the wave effects become important for the lens mass smaller than $\sim 10^8 M_\odot$. For such cases, we compute how accurately we can extract the lens mass from the lensed signal. We consider two simple lens models: the point mass lens and the SIS (Singular Isothermal Sphere). We find that the lens mass can be determined within $\sim 0.1\%$ $[(S/N)/10^3]^{-1}$ for the lens mass larger than $10^8 M_\odot$, where (S/N) is the signal to noise ratio of the unlensed chirp signals. We find that the lensing cross section is an order of magnitude larger than that for the usual strong lensing of light.

1. Introduction

Inspirals and mergers of compact binaries are the most promising gravitational wave sources for the ground based as well as the space based interferometers (TAMA300, LIGO, VIRGO, GEO600 and LISA). If the gravitational waves from coalescing binary pass near massive objects, gravitational lensing should occur in the same way as it does for light. The gravitational lensing of light is usually treated in the geometrical optics approximation, which is valid in all the observational situations (Schneider, Ehlers & Falco 1992; Nakamura & Deguchi 1999). However for the gravitational lensing of gravitational waves, the wavelength is long so that the geometrical optics approximation is not valid in some cases. For example, the wavelength λ of the gravitational waves for the space interferometer is ~ 1 AU which is extremely larger than that of a visible light ($\lambda \sim 1\mu$ m). As shown by several authors (e.g. Bontz & Haugan 1981), if the wavelength λ is larger than the Schwarzschild radius of the lens mass M_L, the diffraction effect is important and the magnification is small. Since the gravitational waves from the compact binaries are coherent, the interference is also important (e.g. Deguchi & Watson 1986).

In this paper, we discuss the gravitational lensing of gravitational waves,

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taking account of the wave effects in the gravitational lensing (this paper is based on Takahashi & Nakamura 2003). We take the coalescence of the super massive black holes (SMBHs) of mass $10^6 M_\odot$ as the sources. SMBH binary is one of the most promising sources for LISA and will be detected with very high signal to noise ratio, $S/N \sim 10^3$ (Bender *et al.* 2000). Since the merging SMBHs events will be detected for extremely high redshift ($z > 5$), the lensing probability is relatively high and hence some lensing events are expected. We consider the two simple lens models; the point mass lens and the SIS (Singular Isothermal Sphere) lens. The wave effects become important for the lens mass smaller than $10^8 M_\odot (f/\mathrm{mHz})^{-1}$ in the case of LISA ($f \sim$ mHz). We calculate the gravitational lensed waveform using the wave optics. Then, we investigate how accurately we can extract the information on the lens object from the gravitational lensed signals detected by LISA using the Fisher-matrix formalism (e.g. Cutler 1998). Following Cutler (1998), we calculate the estimation error for the lens mass. We assume the 1 yr observation before the final merging and consider the lens mass in the range $10^6 - 10^9 M_\odot$. Then the typical time delay between the double images is $10 - 10^4$ sec which is much smaller than 1 yr.

2. Gravitational Lensed Waveform

The gravitational lensed waveform $\tilde{h}^L(f)$ in the frequency domain are given by the product of the amplification factor $F(f)$ and the unlensed waveform $\tilde{h}(f)$ (see Schneider, Ehlers & Falco 1992; Nakamura & Deguchi 1999),

$$\tilde{h}^L(f) = F(f)\ \tilde{h}(f). \tag{1}$$

The amplification factor $F(f)$ depends on the two lens parameters; the lens mass M_L and the source position y which is a position vector of the source divided by the Einstein radius in the source plane.

In Fig.1, we show the amplification factor $|F(f)|$ as a function of w ($= 8\pi M_{Lz} f$) for the fixed source position $y = 0.1, 0.3, 1, 3$ for the point mass lens (left panel) and the SIS lens (right panel). For $w < 1$, the amplification is very small due to the diffraction effect. Since in this case the wave length is so long that the wave does not feel the existence of the lens. For $w > 1$, $|F(f)|$ asymptotically converges to the geometrical optics limit. The oscillatory behavior (in Fig.1) is due to the interference between the double images. We note that even for $y \geq 1$ in SIS model ($y = 3$ in Fig.1) the damped oscillatory behavior appears, although only a single image exists in the geometrical optics limit.

For the unlensed waveform $\tilde{h}(f)$ in Eq.(1), we use restricted post-Newtonian approximation as the in-spiral waveform (Cutler & Flanagan 1994). We assume that the lensed signal $\tilde{h}^L(f)$ is characterized by some unknown parameters γ_i which are ten source parameters (binary masses, spatial position and so on) and

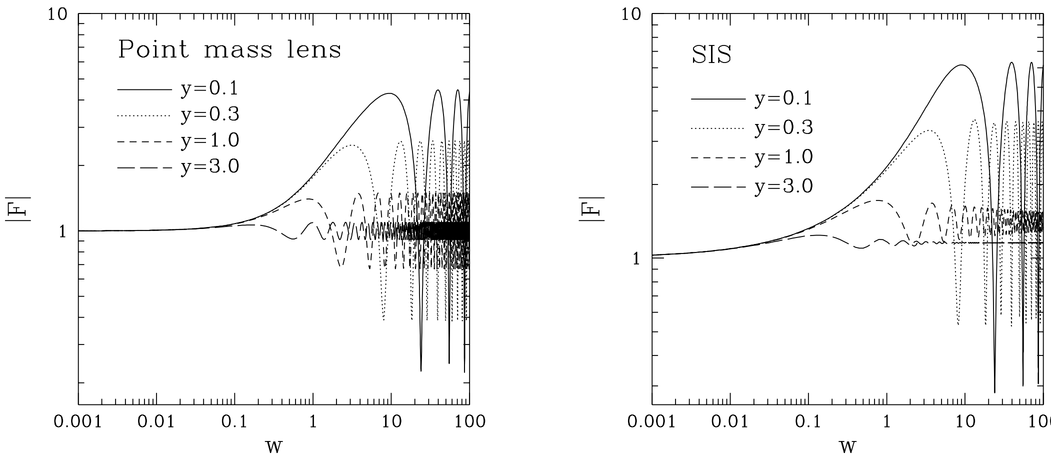

Fig. 1. The amplification factor $|F(f)|$ as a function of w $(= 8\pi M_{Lz} f)$ with the fixed source position $y = 0.1, 0.3, 1, 3$ for the point mass lens (left panel) and the SIS (right panel).

two lens parameters (M_L, y). We use the Fisher-matrix formalism to calculate errors $\Delta\gamma_i$ in the parameter estimation (e.g. Cutler & Flanagan 1994).

3. Results

We show the parameter estimation for the lens mass M_L. We show the results for the SMBH binary with masses $10^6 + 10^6 M_\odot$ at redshift $z = 1$. We assume 1 yr observation of in-spiral phase before final merging.

In Fig.2, the estimation errors for the lens mass ΔM_L are shown for the point mass lens as a function of the lens mass M_L (left panel) and the source position y (right panel). We use the units of $S/N = 10^3$, and the results ΔM_{Lz} scale as $(S/N)^{-1}$. In the left panel, for $M_{Lz} < 10^7 M_\odot$ the estimation errors are relatively large $> 10\%$, since the effect of lensing on the signals is very small due to the diffraction. For $M_{Lz} > 10^8 M_\odot$ the geometrical optics approximation is valid, and the errors converge to a constant in the left panel of Fig.2 The lens mass can be determined up to the accuracy of $\sim 0.1\%[(S/N)/10^3]^{-1}$.

In the right panel of Fig.2, for $y > 1$ the errors are convergent to the geometrical optics limit irrespective of the lens mass. As y increases, the time delay t_d increases, and the geometrical optics limit $(f t_d \gg 1)$ is valid. We note that even for $y > 10$ one can extract the lens information. Thus the lensing cross section $(\propto y^2)$ increases an order of magnitude larger than that for the usual strong lensing of light $(y = 1)$.

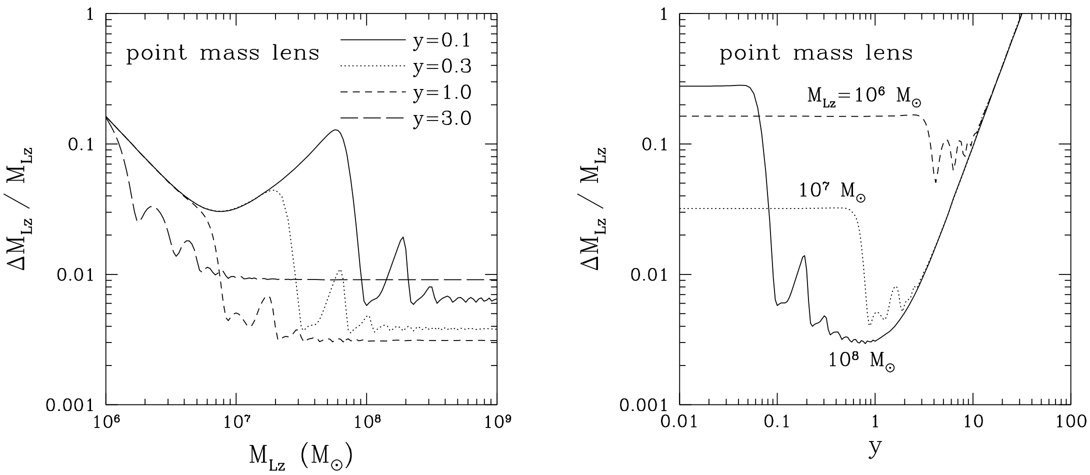

Fig. 2. The estimation error for the lens mass ΔM_L for the point mass lens as a function of M_L (left panel) and y (right panel)

4. Summary

We have discussed the gravitational lensing of gravitational waves from chirping binaries, taking account of the wave effects in gravitational lensing. The SMBH binary is taken as the source detected by LISA, and the two simple lens models are considered: the point mass lens and the SIS model. We calculate how accurately the information of the lens object, its mass, can be extracted from the lensed signal. For the lens mass larger than $10^8 M_\odot$ the lens parameters can be determined within (very roughly) $\sim 0.1\% \, [(S/N)/10^3]^{-1}$. We note that the lensing cross section is order of magnitude larger than that for light.

5. References

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