The Bell-Lucek Mechanism in SNRs and the "Knee" in the Cosmic Ray Spectrum

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Abstract

We use the simplified "box model" description of particle acceleration in supernova remnants to investigate the spectral modifications induced by the Bell-Lucek magnetic field amplification process. In particular we examine whether such models can naturally account for the shape of the "knee" in the observed Galactic cosmic ray spectrum.

1. Introduction

Bell & Lucek [1] and Lucek & Bell [3] have presented numerical simulations suggesting that the conventional process of particle acceleration in shocks bounding young supernova remnants (YSNRs) may result in substantial amplification of the highly tangled magnetic field around the shock. Cas A may well be an example of such a system, because of its unusual strong magnetic field [4]. The Bell-Lucek hypothesis is one of the few suggestions as to how cosmic ray particles at and above the "Knee" could be accelerated in relatively conventional Galactic sources and as such deserves serious consideration.

2. The Bell-Lucek Hypothesis

In the Bell & Lucek process the magnetic field near the shock is highly distorted by the strong particle pressure gradients and wound up to the point where approximate equipartition holds. As a result the effective magnetic field scales with the velocity of the blast wave bounding the YSNR. Using this amplified field to estimate the particle diffusion in the shock neighbourhood obviously allows acceleration to higher energies than in the conventional picture. Detailed estimates and dimensional analysis agree that the maximum particle rigidity is given, to order of magnitude, by the product of the magnetic field strenght B, the shock radius R_{snr} and the velocity of the shock V_{snr}. The increase in cut-off energy is directly proportional to the field amplification which, because of the

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equipartition argument, is of order of the Alfven Mach Number of the shock $\mathcal{M}_{\rm sh}$. This can easily be $\mathcal{M}_{\rm sh} \simeq 10^3$ for a YSNR; acceleration to rigidities of a few 10^{17} V, rather than the 10^{14} V normally estimated, is therefore easily possible.

3. The Box Model

In a box model (*e.g.* Drury et al.[2]), the accelerated particles are assumed to be uniformly distributed throughout a region extending one diffusion length each side of the shock, and to be accelerated upwards in momentum space at the shock itself with an acceleration flux

$$\Phi(p) = \frac{4\pi}{3} p^3 f(p)\,(U_1 - U_2)\,, \tag{1}$$

per unit surface area where U_1 and U_2 are the upstream and downstream velocity and $f(p)$ is the phase space density of the accelerated particles. If the diffusion length upstream is L_1, and that downstream is L_2, then

$$L_1 \approx \frac{\kappa_1(p)}{U_1}, \qquad L_2 \approx \frac{\kappa_2(p)}{U_2}, \tag{2}$$

where κ_1 and κ_2 are the upstream and downstream diffusion coefficients, where we assume Bohm scaling. To a first approximation we assume that both L_1 and L_2 are small relative to the radius of the shock and that we can neglect effects of spherical geometry so that the box volume is simply $A(L_1 + L_2)$ where A is the surface area of the shock. The basic "box" model equation is then simply a conservation equation for the particles in the box; the rate at which the number in the box changes is given by the divergence of the acceleration flux in momentum space plus gains from injection and advection and minus advective losses to the downstream region.

$$\frac{\partial}{\partial t}\left[A(L_1 + L_2) 4\pi p^2 f(p)\right] + A\frac{\partial \Phi}{\partial p} = AQ(p) + AF_1(p) - AF_2(p), \tag{3}$$

where $Q(p)$ is a source function representing injection at the shock (only important at very low energies), F_1 is a flux function representing advection of pre-existing particles into the system from upstream (normally neglected) and F_2 is the flux of particles advected out of the system and carried away downstream. The only complication we have to consider is that the box is time-dependent, with flow speeds, shock area and diffusion lengths all changing. The escaping flux is determined simply by the advection across the downstream edge of the box, that is

$$F_2(p) = 4\pi p^2 f(p) \left(U_2 - \frac{\partial L_2}{\partial t}\right). \tag{4}$$

Using the above equation plus ignoring the $F_1(p)$ term, the box equation becomes:

$$\frac{1}{A}\frac{\partial A}{\partial t}\left(L_1+L_2\right)f+\frac{\partial L_1}{\partial t}f+\left(L_1+L_2\right)\frac{\partial f}{\partial t}+U_1 f+\left(U_1-U_2\right)\frac{p}{3}\frac{\partial f}{\partial p}=\frac{Q}{4\pi p^2}. \quad (5)$$

Partial differential equations of this form always reduce, by the method of characteristics, to the integration of two ordinary equations, one for the characteristic curve in the (p,t) plane

$$\frac{\mathrm{d}\,p}{\mathrm{d}\,t}=\frac{U_1-U_2}{L_1+L_2}\frac{p}{3}, \quad (6)$$

and one for the variation of f along this curve

$$\left(L_1+L_2\right)\frac{\mathrm{d}\,f}{\mathrm{d}\,t}+f\left[\left(L_1+L_2\right)\frac{1}{A}\frac{\partial A}{\partial t}+\frac{\partial L_1}{\partial t}+U_1\right]=\frac{Q}{4\pi p^2}, \quad (7)$$

where apart from the injection momentum we can set $Q=0$.

4. Spectral Modifications in the Sedov-Taylor solution

In order to derive spectral information out of the box model we rewrite equation (7), assuming Bohm scaling ($\kappa\propto p/B$) and writing $\vartheta=L_1/(L_1+L_2)$, as:

$$\frac{\mathrm{d}\,\ln f}{\mathrm{d}\,t}=-\frac{\mathrm{d}\,\ln A}{\mathrm{d}\,t}+\vartheta\frac{\mathrm{d}\,\ln(U_1B_1)}{\mathrm{d}\,t}-\frac{3U_1}{U_1-U_2}\frac{\mathrm{d}\,\ln p}{\mathrm{d}\,t}, \quad (8)$$

which integrates trivially to relate the value of f at the end of one of the characteristic curves, say at the point (p_1,t_1), to the value at the start, say at (t_0,p_0), as follows;

$$\frac{f(t_1,p_1)}{f(t_0,p_0)}=\left(\frac{A(t_1)}{A(t_0)}\right)^{-1}\left(\frac{U_1(t_1)B_1(t_1)}{U_1(t_0)B_1(t_0)}\right)^{\vartheta}\left(\frac{p_1}{p_0}\right)^{-3U_1/(U_1-U_2)}. \quad (9)$$

Furthermore we rewrite equation (6), the characteristic curve in the (p,t) plane, as an equation for kinetic energy, $T=c\sqrt{p^2+m^2c^2}-mc^2\approx cp$, assuming Bohm diffusion ($\kappa=pc^2/3eB$) again:

$$T_1-T_0=\frac{e}{\alpha}\int_{t_0}^{t_1}\left(U_1-U_2\right)U_1B_1\,dt, \quad (10)$$

where α is a numerical factor of order 10. Essentially the equation for the curve relates final energies to starting times: in the case of a a Sedov-Taylor expansion law for the YSNR ($R_{\rm snr}\propto t^{2/5}$,$U_{\rm snr}\propto t^{-3/5}$), $T_1\gg T_0$ and $t_0\ll t_1$, this yields:

$$p_1\propto T_1\propto t_0^{-4/5}, \qquad t_0\propto p_1^{-5/4}. \quad (11)$$

Using the above relation we can translate the different terms on the RHS of equation (9) to additional power-law terms in the final momentum p_1: the first

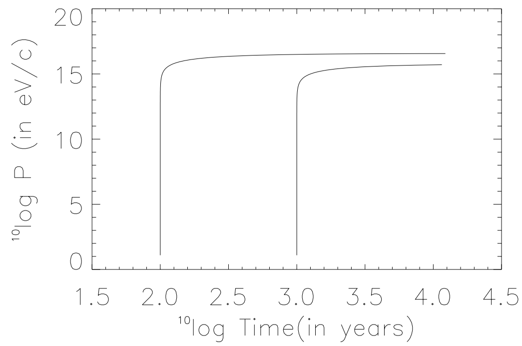

Fig. 1. Example of 2 characteristic curves in the (p, t) plane for a Sedov-Taylor expansion law and a magnetic field which scales with the shock velocity.

term on the RHS of equation (9) translates to a p_1^{-1} factor , the second term on the RHS yields a $p_1^{-3\vartheta/2}$ factor. The second term includes the result from the Bell-Lucek hypothesis, i.e. $B \propto U_{\rm snr}$. For the injection momentum we assume $p_{\rm inj} = p_0 \propto U(t_0) \propto p_1^{3/4}$ together with $f_0 \propto p_0^{-3}$. Finally assuming a strong shock which, using the Rankine-Hugoniot conditions for a non-relativistic fluid, yields $U_1/U_2 = 4$ and $3U_1/(U_1 - U_2) = 4$, we obtain a scaling-law by rewriting equation (9), which yields the particle distribution $f(p_1)$ at a fixed time t_1:

$$f(p_1) \propto p_0^{-3} A(t_0) \left[U_1(t_0) B_1(t_0)\right]^{-\vartheta} \left(\frac{p_1}{p_0}\right)^{-4} \propto p_1^{3/4} p_1^{-1} p_1^{-3\vartheta/2} p_1^{-4}, \qquad (12)$$

the slope is steepened from the canonical value of 4 to

$$4.25 + \frac{3\vartheta}{2} \qquad (13)$$

in the range where this simple analysis is applicable.

5. Conclusion

We have used a simplified "box model" to describe particle acceleration in a Sedov-Taylor supernova remnant where the amplified magnetic field is assumed to scale with the velocity of the blast wave. We have shown that under plausible assumptions this produces a high-energy tail to the spectrum which is slightly steeper than 4 with a transition to the usual spectrum in the "knee" region.

References

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