New Suggested Strategy For Detecting Gravitational Waves

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Abstract

An alternative approach, which is consistent with the conventional approach adopted by GW experimentalists and detectors biulders, is suggested to be used to assure the detection of GW. The proposed approach suugests a procedure to search for the square root of the amplitude of GW in the data of a single detector under certain given condition on the length control systems.

1. Introduction

The question of detecting GW represents one of the fundamental questions and a long standing challenge in Gravitational physics. Most of the main difficulties facing the detection of GW are on the way to be overcome very recently [cf. Saulson (2000), Ando et al. (2001, 2002), Kuroda et al. (2002) and Gonzales (2003)]. Several collaborations are established to ensure that the recorded signals by different detectors, with uncorrelated interinsic and stochastic noises, are due to passing of GW through them [cf. Prodi (2000), Astone (2002), Sigg (2002), Acernese (2002) and Willke (2002)]. In this paper, an alternative approach; which is consistent with the conventional approach adopted by the existing and future detectors biulders; is suggested.

2. Philoshophy of the approach

Since in any ground based GW experiment; the coordinate system at which the bar (interfermoter's arm) is fixed, coincides with the coordinate system at which the measuring apparatus of the tiny changes (δx) in its proper (physical) length l_0; then due to the special theory of relativity, the known relation between l_0 and the relative length l; $l = l_0\sqrt{1-\frac{v^2}{c^2}}$; becomes $l = l_0$. Hence, the physical (proper) length of a bar (arm) in any ground based GW experiment can be treated as a scalar from the special relativistic point of view.

From the general relativistic point of view, the proper time (interval) of a bar (arm) having two end points $(x_2^0, x_2^1, x_2^2, x_2^3)$ and $(x_1^0, x_1^1, x_1^2, x_1^3)$; situated in a four dimensional general coordinate system (K); is defined as:

$$S = [g_{\mu\nu}(x_2^\mu - x_1^\mu)(x_2^\nu - x_1^\nu)]^{\frac{1}{2}} = [g_{00}(x_2^0 - x_1^0)(x_2^0 - x_1^0) + g_{ij}(x_2^i - x_1^i)(x_2^j - x_1^j)]^{\frac{1}{2}} \quad (1)$$

where $g_{\mu\nu}$ is the metric tensor representing the gravitational background in which a bar (arm) is swimming freely and $i, j = 1, 2, 3$. While it is defined in another

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coordinate system (K') as:

$$S' = [g'_{\mu\nu}(x'^{\mu}_2 - x'^{\mu}_1)(x'^{\nu}_2 - x'^{\nu}_1)]^{\frac{1}{2}} = [g'_{00}(x'^{0}_2 - x'^{0}_1)(x'^{0}_2 - x'^{0}_1) + g'_{ij}(x'^{i}_2 - x'^{i}_1)(x'^{j}_2 - x'^{j}_1)]^{\frac{1}{2}} \tag{2}$$

Since in any GW experiment, both systems having the bar (arm) and the measuring apparatuses of δx and the time δx^0 are coincident, then the general coordinate transformation between both systems K, K' is given by $x^{\mu} = x'^{\mu}$ and $g_{\mu\nu}(K) = g'_{\mu\nu}(k')$ as well. Besides that, in the case of a single detector, $x^0 = x'^0$ and $g_{00} = g'_{00}$ then one can deduce that the pysical length of a bar (arm) reads as:

$$L = [g_{ij}(x^i_2 - x^i_1)(x^j_2 - x^j_1)]^{\frac{1}{2}} = [g'_{ij}(x'^i_2 - x'^i_1)(x'^j_2 - x'^j_1)]^{\frac{1}{2}} \tag{3}$$

If a bar (arm) situated parallel to x- axis, then L reads as:

$$L = [g_{11}(x^1_2 - x^1_1)(x^1_2 - x^1_1)]^{\frac{1}{2}} \tag{4}$$

Hence, the physical length of a bar (arm); which is situated parallel to x- axis; $L(\delta x, \delta t)$ can be treated as a scalar function of δx and δt; where δx is the tiny changes in its length due to different sources of noise during time interval δt, or equivalently due to any signal having a frequency band $\delta\omega$ ($\delta\omega = \frac{1}{\delta t}$).

3. Methodology of the approach

It is possible to define; in general; the magnitude of the gradient of the physical length $L(\delta x^{\mu})$; in the presence of any gravitational field described by the covariant metric tensor $g^{\mu\nu}$; as follows:

$$M_L = (g^{\mu\nu} L_{\mu} L_{\nu})^{\frac{1}{2}} \tag{5}$$

where $L_{\mu} = \frac{\partial L}{\partial(\delta x^{\mu})}$ and $\mu = 0, 1, 2, 3$. Hence it is natural to define a covariant function G_L as follows:

$$G_L = \frac{dM_L}{dS} = \frac{1}{M_L} g^{\mu\nu} L_{\mu;\sigma} L_{\nu} U^{\sigma} \tag{6}$$

where S is the proper time, $L_{\mu;\sigma}$ is the covariant derivative of L_{μ} with respect to (δx^{μ}) and $U^{\sigma} = \frac{d(\delta x^{\sigma})}{dS}$ [cf. Melek (2002)]. If a bar (arm) is situated parallel to the x- axis, then $L(\delta x^{\mu})$ will be function of δx and δt; i.e. $L(\delta x, \delta t)$. In the case of studying the effect of the GW on the magnitude of the gradient of the physical length of a bar (arm), then $g^{\mu\nu}$ is calculated using the line element:

$$dS^2 = dt^2 - (1 + h(t, z))dx^2 - (1 - h(t, z))dy^2 - dz^2 \tag{7}$$

Therefore, using the line element (7) and carrying out the necessary manipulation of the expression (6), the function $G_L$can be expressed as [Melek (2002)]:

$$G_L = \frac{(L_0^2 - L_1^2)^{\frac{1}{2}}}{M_L} \frac{d(L_0^2 - L_1^2)^{\frac{1}{2}}}{dS} + \frac{hL_1^2}{M_L}[\frac{1}{L_1}(\frac{dL_1}{dS}) + \frac{1}{2h}(\frac{dh}{dS})] \tag{8}$$

where

$$M_L = (L_0^2 - L_1^2 + hL_1^2)^{\frac{1}{2}}, \tag{9}$$

$$\frac{dL_1}{dS} = \frac{\partial L_1}{\partial(\delta t)}\frac{d(\delta t)}{dS} + \frac{\partial L_1}{\partial(\delta x)}\frac{d(\delta x)}{dS} \tag{10}$$

and

$$\frac{dh}{dS} = \frac{\partial h}{\partial t}\frac{dt}{dS} + \frac{\partial h}{\partial t}\frac{dt}{dS} \tag{11}$$

4. Theoretical Considerations

It is worth to notice in the expression (8) that the theoretical condition $L_0^2 = L_1^2$ or equivalently:

$$L_1 = \pm L_0 \tag{12}$$

leads to the following expression of G_L

$$G_L = \sqrt{h}L_1[\frac{1}{L_1}\frac{dL_1}{dS} + \frac{1}{2h}\frac{dh}{dS}] \tag{13}$$

or equivalently

$$G_L = \pm\sqrt{h}L_0[\frac{1}{L_0}\frac{dL_0}{dS} + \frac{1}{2h}\frac{dh}{dS}] \tag{14}$$

It is possible to reinterpret the condition (12) as a constraint on the variation of L with respect to δx and its variation with respect to the frequency band $\delta\omega$ ($\delta\omega = (\delta t)^{-1}$) causes the δx in the length L, as follows:

$$\frac{\partial L}{\partial(\delta x)} = \pm(\delta\omega)^2\frac{\partial L}{\partial(\delta\omega)} \tag{15}$$

A decisive question should be raised about the experimental viability of the constraint (15). This question may be formulated as follows:

"Whether the length control systems in the existing detectors may fulfill the constraint (15) or not?"

[cf. the senstivity curves of TAMA and LIGO in Ando et al. (2001, 2002) and in Gonzalez (2003a)]. The author hopes that the definite answer of this question will come from the GW experimentalists and the detectors biulders.

5. Searching for $\sqrt{h}$ of GW in the data of a single detector

Using the condition (12) in the original defenition of G_L (6), one can get:

$$G_L = \frac{d\sqrt{h}L_0}{dS} \tag{16}$$

Therefore the expression (14) can be expressed as:

$$\frac{\Delta(\sqrt{h}L_0)}{\sqrt{h}L_0} = \pm[\frac{\Delta L_0}{L_0} + \frac{\Delta h}{2h}] \tag{17}$$

Since $\frac{\Delta L}{L} = \frac{1}{2}h$, then $\frac{\Delta h}{h} = \frac{\Delta(\frac{\Delta L}{L})}{(\frac{\Delta L}{L})}$, and hence the expression (17) can be expressed as:

$$\frac{\Delta(\sqrt{h}L_0)}{\sqrt{h}L_0} = \frac{\Delta[\sqrt{h}(\delta\omega)^2(\frac{\Delta L}{\Delta(\delta\omega)})]}{[\sqrt{h}(\delta\omega)^2(\frac{\Delta L}{\Delta(\delta\omega)})]} = \pm[\frac{\Delta L_0}{L_0} + \frac{\Delta(\frac{\Delta L}{L})}{2(\frac{\Delta L}{L})}] \tag{18}$$

The right hand side of the expression (18) can be calculated from the output of the detector (ΔL)'s at different δt's or equivalently at different $\delta\omega$'s during its operational period. Therefore, the calculation of the right hand side of the expression (18) will give the fluctuations in the quantity $\sqrt{h}(\delta\omega)^2(\frac{\Delta L}{\Delta(\delta\omega)})$, from which one can get a trace (finger print) of $\sqrt{h}$, by using one of the widely accepted data analysis schemes.

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