One-Armed Spiral Instability in Differentially Rotating Stars

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Abstract

We investigate the dynamical instability of the one-armed spiral $m = 1$ mode in differentially rotating stars by means of hydrodynamical simulations in Newtonian gravitation. We find that both a soft equation of state and a high degree of differential rotation in the equilibrium star are necessary to excite a dynamical m=1 mode as the dominant instability at small values of the ratio of rotational kinetic to potential energy, $T/|W|$. We find that this spiral mode propagates outward from its point of origin near the maximum density at the center to the surface over several central orbital periods. An unstable $m = 1$ mode triggers a secondary $m = 2$ bar mode of smaller amplitude, and the bar mode can excite gravitational waves. As the spiral mode propagates to the surface it weakens, simultaneously damping the emitted gravitational wave signal. This behavior is in contrast to waves triggered by a dynamical $m = 2$ bar instability, which persist for many rotation periods and decay only after a radiation-reaction damping timescale.

1. Introduction

Stars in nature are usually rotating and may be subject to nonaxisymmetric rotational instabilities. An exact treatment of these instabilities exists only for incompressible equilibrium fluids in Newtonian gravity [2, 5]. For these configurations, global rotational instabilities may arise from non-radial toroidal modes $e^{im\varphi}$ (where $m = \pm 1, \pm 2, \ldots$ and φ is the azimuthal angle).

For sufficiently rapid rotation, the $m = 2$ bar mode becomes either *secularly* or *dynamically* unstable. The onset of instability can typically be identified with a critical value of the non-dimensional parameter $\beta \equiv T/|W|$, where T is the rotational kinetic energy and W the gravitational potential energy. Uniformly rotating, incompressible stars in Newtonian theory are secularly unstable to bar-mode formation when $\beta \geq \beta_{\rm sec} \simeq 0.14$. This instability can grow only in the presence of some dissipative mechanism, like viscosity or gravitational radiation, and the associated growth timescale is the dissipative timescale, which is usually much longer than the dynamical timescale of the system. By contrast, a dynamical instability to bar-mode formation sets in when $\beta \geq \beta_{\rm dyn} \simeq 0.27$. This

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instability is independent of any dissipative mechanisms, and the growth time is the hydrodynamic timescale.

Determining the onset of the dynamical bar-mode instability, as well as the subsequent evolution of an unstable star, requires a fully nonlinear hydrodynamic simulation. Simulations performed in Newtonian gravity have shown that $\beta_{\rm dyn}$ depends only very weakly on the stiffness of the equation of state. Once a bar has developed, the formation of a two-arm spiral plays an important role in redistributing the angular momentum and forming a core-halo structure. Both $\beta_{\rm dyn}$ and $\beta_{\rm sec}$ are smaller for stars with high degree of differential rotation. Simulations in relativistic gravitation have shown that $\beta_{\rm dyn}$ decreases with the compaction of the star, indicating that relativistic gravitation enhances the bar mode instability. In order to efficiently use computational resources, most of these simulations have been performed under certain symmetry assumptions (e.g. π-symmetry), which do not affect the growth of the $m = 2$ bar mode, but which suppress any $m = 1$ modes.

Recently, Centrella et al. [1] reported that such $m = 1$ "one-armed spiral" modes are dynamically unstable at surprisingly small values of $T/|W|$. Centrella et al. [1] found this instability in evolutions of highly differentially rotating equilibrium polytropes with polytropic index $n = 3.33$. Typically, these equilibria have a "toroidal" structure, so that the maximum density is not located at the geometric center but rather on a toroid rotating about the center.

The purpose of this paper is to study further the conditions under which a dynamical $m = 1$ instability is excited. We vary both the polytropic index, i.e. the stiffness of the equation of state, and the degree of differential rotation to isolate their effects on the instability. Since the onset of rotational instabilities is often characterized by β we keep this value approximately fixed in our sequences. We find that a soft equation of state and a high degree of differential rotation are both necessary to dynamically excite the $m = 1$ mode at the small value of $\beta = 0.14$ chosen in this paper. We find that a toroidal structure is not sufficient to trigger the $m = 1$ instability, but our findings suggest that a toroidal structure may be necessary.

This paper is organized as follows. We discuss our numerical results in § 2, and briefly summarize our findings in § 3. Throughout this paper we use gravitational units * with $G = c = 1$ and adopt Cartesian coordinates (x, y, z). A more detailed discussion is presented in Ref. [4].

*Since we adopt Newtonian gravity in this paper, the speed of light only enters in the gravitational waveforms.

2. Numerical Results

We parametrize the stiffness of the equation of state by varying the polytropic index n between $n = 3.33$ and $n = 2$. In this sequence we keep the degree of differential rotation (i.e. Ω_c/Ω_{eq}) fixed, and adjust the overall rotation rate (parametrized by the ratio of the polar to equatorial radius R_p/R_{eq}) so that the value of $T/|W|$ remains very close to 0.144 (as for Model I (a) in Table 2 of Ref. [4]). We list our four different Models II in Table 3 of Ref. [4].

Figure 9 of Ref. [4], where we plot the dipole diagnostic D as a function of time, clearly shows that an $m = 1$ instability is excited in Models II (b) and II (c) in addition to Model II (d). We find that D starts decreasing immediately after reaching a maximum. This is a consequence of the star rearranging its density profile, and of the spiral arm propagating outward to lower density regions. Model II (a), however, which has the most pronounced toroidal structure, remains stable.

In Fig. 1., we show the maximum density as a function of time. The maximum density slowly increases in all cases due to dissipation of differential rotation. Once the one-armed spiral forms in Models II (b) and II (c), however, this increase is much more rapid, which indicates again that the unstable mode rearranges the matter in the star and destroys the toroidal structure.

We show the gravitational wave signal emitted from Models II in Fig. 13 of Ref. [4]. As consistent with the diagnostic D, the gravitational wave signal emitted by the one-armed spiral mode does not persist over many rotational periods, and instead decays fairly rapidly after it has been excited. This characteristic is very different from what has been found for $m = 2$ bar mode instabilities (compare § 3.2 of Ref. [4]). We also find that the maximum wave amplitude is much smaller than can be found for configurations unstable to a pure bar mode (compare Fig. 3 of Ref. [4]) as gravitational radiation requires a quadrupole distortion and the $m = 2$ perturbation in Models II is only being excited as a lower-amplitude harmonic of the $m = 1$ mode.

3. Summary

We have studied the conditions under which Newtonian, differentially rotating stars are dynamically unstable to an $m = 1$ one-armed spiral instability, and found that both soft equations of state and a high degree of differential rotation are necessary to trigger the instability. For sufficiently soft equations of state and sufficiently high degrees of differential rotation we found that stars are dynamically unstable even at the small values of $T/|W| \sim 0.14$ considered in this paper.

While we find that a toroidal structure alone is not sufficient for the $m = 1$ instability, all the models that are unstable do have a toroidal structure, suggesting that this may be a necessary condition. The growing $m = 1$ mode redis-

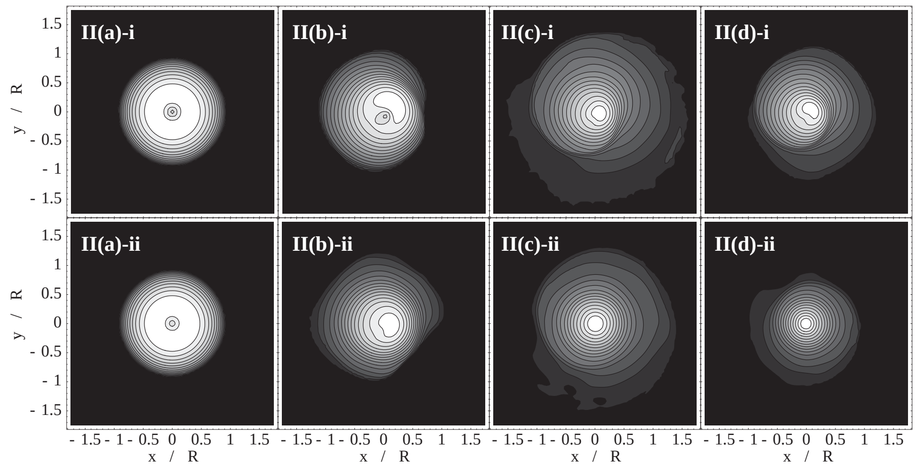

Fig. 1. Intermediate ($t_{\rm int} \approx 0.7 t_{\rm fin}$) and final density contours in the equatorial plane for Models II (See Ref. [4] in details).

tributes the matter in the unstable star and destroys the toroidal structure after a few central rotation periods.

Quasi-periodic gravitational waves emitted by stars with $m = 1$ instabilities have smaller amplitudes than those emitted by stars unstable to the $m = 2$ bar mode. For $m = 1$ modes, the gravitational radiation is emitted not by the primary mode itself, but by the $m = 2$ secondary harmonic which is simultaneously excited, albeit at a lower amplitude. Unlike the case for bar-unstable stars, the gravitational wave signal does not persist of many periods, but instead is damped fairly rapidly in most of the cases we have examined.

We have plotted typical wave forms for stars unstable to $m = 2$ bar modes in Fig. 3 of Ref. [4] and for stars unstable to one-armed spiral $m = 1$ modes in Figs. 13 and 18 of Ref. [4]. Characteristic wave frequencies $f_{\rm GW}$ are seen to be $\sim P_{\rm c}^{-1} \sim \Omega_{\rm c}$, and are considerably higher than $\Omega_{\rm eq} \sim (M/R^3)^{1/2}$ due to appreciable differential rotation. For supermassive stars ($M > 10^5 M_\odot$) the amplitudes and frequencies of these waves fall well within the detectable range of LISA [3].

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