Timescale Analysis of Spectral Time Lags

Ti-Pei Li[1,2]
(1) Dept. of Phys. and Center for Astrophys., Tsinghua University, Beijing
(2)Particle Astrophysics Center, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing

Abstract

Understanding the energy dependence of arrival times of observed high energy particles or photons is of importance in studying cosmic ray physics and high energy astrophysics. Conventional cross-correlation technique has limited resolution in time lag detection and can not be used to get timescale spectra of spectral lags. The Fourier spectral technique often fails in the region of high Fourier frequencies (short time scales). A modified cross-correlation algorithm is introduced in this paper. With the modified cross-correlation function, the sensitivity and precision of detecting spectral time lags can be significantly improved and timescale spectrum of time lag can be derived.

1. Modified Cross-Correlation Function

The time domain method for studying spectral lags is based on the correlation analysis. For two counting series $x(t_i),\ y(t_i)$ (or $x(i),\ y(i)$), the observed counts in the corresponding energy band in the time interval (t_i, t_{i+1}) with $t_i = (i-1)\Delta t$, the cross-correlation function (CCF) of the zero-mean time series is usually defined as

$$\mathrm{CCF}(k) = \sum_i u(i)v(i+k)/\sigma(u)\sigma(v) \quad (k = 1, \pm 1, \cdots) \ , \tag{1}$$

where $u(i) = x(i) - \bar{x}, v(i) = y(i) - \bar{y}$, $\sigma^2(u) = \sum_i [u(i)]^2$ and $\sigma^2(v) = \sum_i [v(i)]^2$. With CCF the time lag can be defined as $\Lambda = k_m \Delta t$ where $\mathrm{CCF}(k)/\mathrm{CCF}(0)$ has maximum at $k = k_m$. The traditional cross-correlation method can give only a single time lag Λ and it fails to calculate any time lag shorter than the time step Δt. Observed intensity variations are usually produced by various processes with different time scales and different spectral lags. In studying a complex process, only a time lag Λ is not enough, we need to know time lags at different timescales, i.e. the timescale spectrum $\Lambda(\Delta t)$. From the cross Fourier spectrum one can derive a lag spectrum, a distribution of time lags over Fourier frequencies. But as the Fourier technique is powerless for detecting the variation power of a stochastic

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process at high frequencies[1], the Fourier analysis method is also powerless for detecting the time lags at high frequencies (short timescales).

A modified cross-correlation technique is proposed [2,3] and further improved in this paper. Let $\{x(i;\delta t)\}$ and $\{y(i;\delta t)\}$ be two lightcurves observed simultaneously in two energy bands with time resolution δt. From $\{x(\delta t)\}$ and $\{y(\delta t)\}$, we can construct two lightcurves $\{x_m(\Delta t)\}, \{y_m(\Delta t)\}$ with the time step $\Delta t = M_{\Delta t}\delta t$ and phase parameter m

$$\begin{aligned} x_m(i;\Delta t) &= \sum_{j=(i-1)M_{\Delta t}+m}^{iM_{\Delta t}+m-1} x(j;\delta t)\ , \\ y_m(i;\Delta t) &= \sum_{j=(i-1)M_{\Delta t}+m}^{iM_{\Delta t}+m-1} y(j;\delta t)\ . \end{aligned} \tag{2}$$

The modified cross-correlation function can be defined as

$$\mathrm{MCCF}(k;\Delta t) = \frac{1}{M_{\Delta t}} \sum_{m=1}^{M_{\Delta t}} \sum_i u_m(i;\Delta t) v_{m+k}(i;\Delta t)/\sigma(u)\sigma(v)\ , \tag{3}$$

where $u_m(i;\Delta t) = x_m(i;\Delta t) - \bar{x}_m(\Delta t)$, $v_m(i;\Delta t) = y_m(i;\Delta t) - \bar{y}_m(\Delta t)$. The procedure of calculating a modified cross-correlation coefficient is schematically illustrated by Figure 1. The time lag of band 2 relative to band 1 on timescale Δt

$$\Lambda(\Delta t) = k_m \delta t \tag{4}$$

where k_m let

$$\mathrm{MCCF}(k = k_m;\Delta t)/\mathrm{MCCF}(0;\Delta t) = \max \tag{5}$$

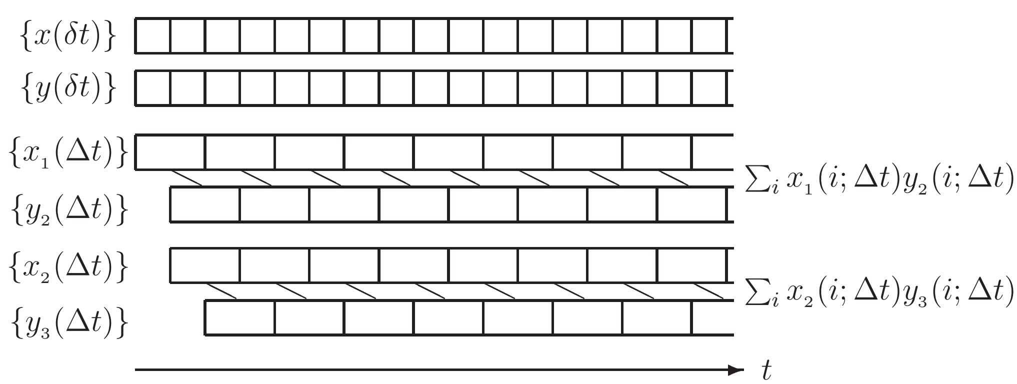

Fig. 1. MCCF of two observed time series $\{x(\delta t)\}$ and $\{y(\delta t)\}$ at a time lag $\tau = \delta t$ ($\tau = k\delta t$, $k = 1$) and on a timescale $\Delta t = 2\delta t$. $\mathrm{MCCF}(k=1;\Delta t) = \frac{1}{2}[\sum_i x_1(i;\Delta t)y_2(i;\Delta t) + \sum_i x_2(i;\Delta t)y_3(i;\Delta t)]$

To compare the above MCCF technique of estimating time lags with the traditional CCF technique and Fourier analysis, we produce two photon event

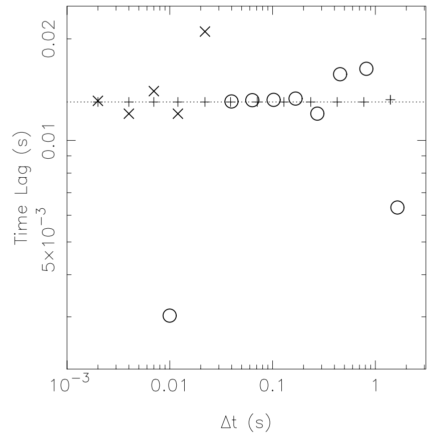

Fig. 2. Time lag vs. time scale of two white noise series with 13 ms time lag shown by the dotted horizontal line. *Cross* – CCF lag; *Circle* – lag from Fourier analysis; *Plus* – MCCF lag by Eq.(3).

series of length 1000 s with a known time lag between them. The series 1 is a white noise series with average rate 200 cts s^{-1} and series 2 consists of the same events in series 1 but each event time is delayed 13 ms. Besides the signal photons mentioned above, the two series are given independent additional noise events at average rate 300 cts s^{-1}. By binning the two event series, two light curves with time resolution $\delta t = 1$ ms are produced. We make time lag analyzes at timescales Δt from 1 ms to 2 s for the two lightcurves by CCF, MCCF and Fourier analysis techniques separately (in Fourier analysis we use Fourier cross spectrum with 1 ms light curves and 4096-point FFT and take Fourier frequency $f = 1/\Delta t$), the results are shown in Figure 2. For the timescale region of Δt shorter or approximately equal to the magnitude of the true lag 0.013 s the CCF works, where MCCF can provide more reliable results with better accuracy. The circles in Fig. 2 indicate the Fourier lags, the Fourier analysis can not give any meaningful result for the short timescale region of $\Delta t < 0.3$ s (or high frequency region of $f > 30$ Hz).

2. Application

The MCCF is particularly useful in studying transient processes. The BATSE detector on the Compton Gamma-Ray Observatory have discovered an unexplained phenomenon: a dozen intense flashes of hard X-ray and gamma-ray photons of atmospheric origin (TGFs)[4]. As all the observed TGFs were of short duration (just a few milliseconds), it is difficult to study their temporal property by conventional techniques. With the aid of MCCF, Feng et al.[5] reveal that for all the flashes with high signal to noise ratio γ-ray variations in the low energy band of 25 - 110 keV relative to the high energy band of $>$ 110 keV are always late in the order of $\sim 100\ \mu$s in the timescale region of $6 \times 10^{-6} - 2 \times 10^{-4}$ s and pulses are usually wide. The above features of energy dependence of time profiles observed in TGFs support models that TGFs are produced by upward explosive electrical discharges at high altitude.

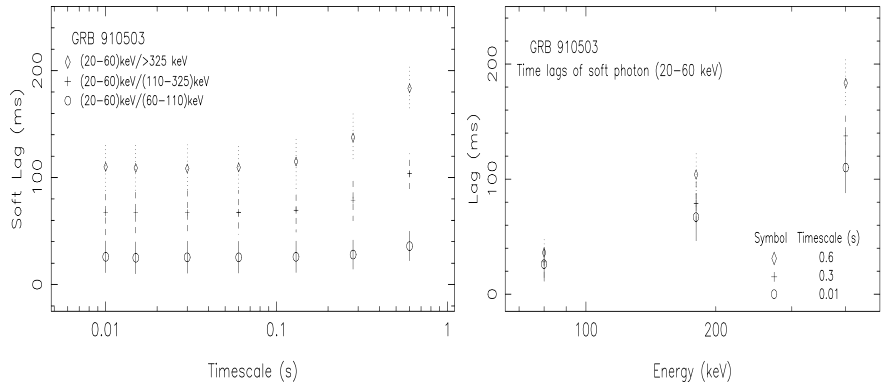

Fig. 3. Soft time lags of GRB 910503 measured by MCCF. *Left panel*: Timescale spectra of time lag. *Circle* – (20-60)keV vs. (60-100)keV; *Plus* – (20-60)keV vs. (110-325)keV; *Diamond* – (20-60)keV vs. $>$ 325keV. *Right panel*: Time lag of 20-60 keV photons vs. energy of hard photons. *Circle* – timescale 0.01 s; *Plus* – timescale 0.3 s; *Diamond* – timescale 0.6 s.

Efforts have been made to measure the temporal correlation of two GRB energy bands by the CCF technique. The CCF technique has no necessary sensitivity to make timing analysis for weak events. For strong bursts the DISCSC data in BATSE database, 4-channel light curves with 64 ms time resolution, are usually analyzed, but it fails with the traditional ACF and CCF in the case that the existed spectral lags comparable or smaller than 64 ms whatever how strong the burst is. The BATSE Time-to-Spill (TTS) data record the time intervals to accumulate 64 counts in each of four energy channels. The TTS data have fine time resolution than 64 ms of DISCSC data when the count rate is above 1000 cts s^{-1}. The TTS data can be binned into equal time bins with a resolution of $\delta t \sim 10$ ms and our simulations show that from the derived lightcurves the temporal and spectral properties with the time resolution δt can be reliably studied with MCCF for typical GRBs recorded by BATSE. As an example, the left panel of Figure 3 shows the soft lag spectrum of GRB 910503 detected by BATSE. Most lags in this figure are less than or approximately equal to related time scales, that can not be detected by CCF. From the MCCF lag spectra, we can further derive the energy dependence of soft lag at different timescales, shown in the right panel of Figure 3.

1. Li T.P., Muraki Y. 2002, ApJ 578, 374
2. Li T.P., Feng Y.X., Chen L. 1999, ApJ 521, 789
3. Li T.P. 2001, Chin. J. Astron. Astrophys. 1, 313
4. Fishman G. J. et al. 1994, Science, 264, 1313
5. Feng H., Li T.P., Wu M., Zha M. & Zhu Q.Q. 2002, Geophys. Res. Lett., 29, 6