The Knee in the Energy Spectrum of Cosmic Rays in the Framework of the Poly-Gonato and Diffusion Models

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Abstract

Results of an analysis of the cosmic-ray energy spectrum using the poly-gonato and diffusion models are presented. Both models produce similar energy spectra and one may conclude that the results are quite consistent with each other. This may indicate that the knee in the cosmic-ray energy spectrum is caused by diffusion processes in the Galaxy.

1. Introduction

Although the knee in the energy spectrum of primary cosmic rays (PCR) at $\approx (3-5)\cdot 10^{15}$ eV was firstly reported more than 40 years ago, [6] its nature is still far from being ultimately established. Up to now, the experiments in the energy region of the knee have been performed with ground-based detector systems investigating extensive air showers (EAS). As these studies provide no direct measurements of PCR fluxes, the reconstruction of the primary energy spectrum from experimental data is a considerable challenge. Hence, recent results of the KASCADE array [9] for energy spectra of individual nuclear groups present a major achievement in the field.

In [2] a phenomenological model, named poly-gonato (Greek "many knees") model, was developed to link the results from direct and indirect measurements. Using this model it proved possible to extrapolate energy spectra of individual elements measured directly to super-high energies. After some slight renormalization of the energy spectra obtained with different EAS arrays the poly-gonato model describes successfully the all-particle energy spectrum as a result of subsequent cut-offs for individual elements (a cut-off proportional to their nuclear charge Z is assumed). Moreover, the primary mass composition extrapolated from energies below 10^{14} eV to super-high energies may be considered as compatible with results derived from EAS experiments and the predictions of the poly-gonato model for individual energy spectra present the experimental results of the KASCADE

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array reasonably well.

So it would be proper to search for some theoretical foundations for the poly-gonato model. The knee may be due to cosmic-ray acceleration and/or propagation. But as there is no accomplished theory of these processes (even for the energy region below the knee) we presently consider only one possibility and try to connect the poly-gonato model with predictions of modern diffusion theory taking into account the drift of cosmic rays in the large scale regular magnetic field (the so-called Hall diffusion) [4][5][7]. If the knee is basically the result of the cosmic-ray propagation in our Galaxy, then the steepening of the PCR energy spectrum stems from the increasing leakage of cosmic rays from the Galaxy when the influence of the Hall diffusion becomes dominant.

2. Basic assumptions

In the region of interest (at energies $E > 10^{15}$ eV) the energy spectrum for cosmic-ray particles with charge Z takes the form [2]

$$\frac{d\Phi_Z}{dE}(E) = \Phi_Z^0 E^{\gamma_Z} \left[1 + \left(\frac{E}{E_Z}\right)^{\epsilon_c}\right]^{\frac{\gamma_c - \gamma_Z}{\epsilon_c}} . \tag{1}$$

where the absolute flux Φ_Z^0 and the spectral index γ_Z define a power law before the knee; γ_c and ϵ_c characterize the change in the spectrum at the cut-off energy E_Z. γ_c and ϵ_c are assumed to be independent on Z. As was shown in [2] it is also possible to use a constant difference $\Delta\gamma$ between the spectral indices before and after the knee. This ansatz leads to

$$\frac{d\Phi_Z}{dE}(E) = \Phi_Z^0 E^{\gamma_Z} \left[1 + \left(\frac{E}{E_Z}\right)^{\epsilon_c}\right]^{\frac{-\Delta\gamma}{\epsilon_c}} . \tag{2}$$

We assume also the dependence $E_Z = ZE_p$ where E_p is the cut-off energy for protons. The all-particle spectrum is obtained by summation over all elements. According to the analysis carried out in [2] there are no statistically significant differences between values of parameters restored when one uses (2) instead of (1).

The diffusion and drift of cosmic rays are considered in the framework of the model adopted in [5]. This model combines Rand-Kulkarni's structure of the regular magnetic field in the Galactic plane [8] and an extended halo as in [7]. Cosmic-ray sources are distributed in the disk with a thickness $2h_s = 400$ pc. The regular magnetic field coincides with Rand-Kulkarni's field in the disk and incorporates a large halo with symmetric or antisymmetric configuration of the magnetic field. The sinusoidal boundary provides necessary signs of the field. The cosmic-ray transport is described by the following equation for the particle

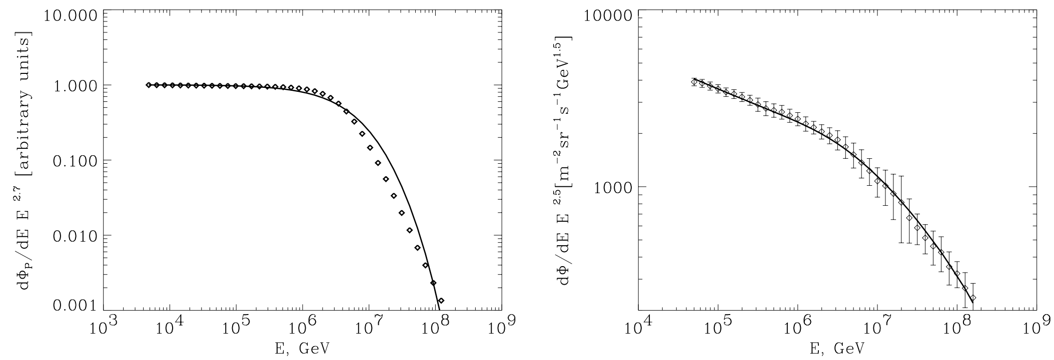

Fig. 1. Left: proton spectrum (◇ – poly-gonato model, solid curve – diffusion model with the radial distribution of sources following the law $Q(r) \sim \delta(r - 4\ \text{kpc})$). Right: ◇ – average all-particle spectrum [2] and diffusion model for elements from H to U according to [2].

concentration $N(r, z)$ (the dependence on E is omitted):

$$\left[-\frac{1}{r}\frac{\partial}{\partial r}rD_{\perp}\frac{\partial}{\partial r} - \frac{\partial}{\partial z}D_{\perp}\frac{\partial}{\partial z} - \frac{\partial}{\partial z}(D_A)\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}(rD_A)\frac{\partial}{\partial z}\right]N(r, z) = Q(r, z) \quad (3)$$

where $Q(r, z)$ is the source term, $D_{\perp}$ and D_A are transverse and Hall diffusion coefficients respectively. Equation (3) is written in a cylindrical frame of reference (r, z), the dependence on ϕ is neglected due to the dominance of the toroidal field component. When solving equation (3) for different Z one must take into account that $D_{\perp} \propto (E/Z)^m$ and $D_A \propto (E/Z)$. Floating boundary conditions are applied in order to calculate $N(r, z)$ with appropriate accuracy near the Galactic plane. Equation (3) is valid up to energies $\geq 10^{17}$ eV and the detailed solution technique may be found in [3].

3. Results of calculations

We have considered a number of variants differing by regular magnetic field configurations, by values of m in $D_{\perp}(E)$ dependence and also by an effective halo height h_{eff}. The latter is defined as follows [7]:

$$h_{eff}^{-1} = (\partial N/\partial z)/N \quad (4)$$

where the concentration and its derivative are taken at $z = 0$. Values of m and h_{eff} are chosen (for any adopted field configuration) to optimize the agreement between the calculated spectra and the predictions of the poly-gonato model.

Fig. 1 (left) presents the energy spectrum of protons calculated in the framework of the diffusion theory and the corresponding spectrum obtained with the poly-gonato model. It may be seen that sufficiently good agreement could be achieved and, therefore, both models should produce similar all-particle spectra.

Indeed, taking the intensities of different elements and their spectral indices according to [2] we obtain quite a good fit of the calculated all-particle energy spectrum to experimental data derived from EAS observations (see Fig. 1 (right)). Extragalactic protons are not taken into account and so the data for energies $\geq 2 \cdot 10^{17}$ eV are not shown. We would like to point out also that the experimental all-particle energy spectrum might be reproduced quite satisfactory even if the predictions of the diffusion and poly-gonato models for individual elements would agree worse.

Equation (3) used to analyze the cosmic-ray transport ignores interactions of nuclei and this assumption, of course, is not exactly valid as these interactions do exist. Some amount of protons and light nuclei should be produced due to nuclear collisions and so the PCR spectrum derived using the diffusion model should display a more complicated behavior after the knee.

4. Conclusion

The energy spectra adopted in the poly-gonato model are almost the same as predicted by modern diffusion theory. So it is quite possible to draw the conclusion that the knee in the PCR energy spectrum may be attributed to diffusion in our Galaxy. But this notion implies that the energy spectra at the sources have to be essentially steeper below the knee as it follows from the standard theory of shock acceleration (see [1]). Indeed, the spectral index at the sources should be about $\gamma_s \approx \gamma_{obs} - m$ and m is close to $0.2 - 0.3$. Though there are no ultimate arguments in favour of the validity of the shock acceleration mechanism some dissatisfaction exists. We would not like to state definitely that the knee in the PCR spectrum is due to the cosmic-ray diffusion only. Probably the adequate reason for the knee should be searched for considering combined influence of acceleration and diffusion processes.

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