# About EAS Inverse Problem

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## Abstract

It is shown that using the observed extensive air shower (EAS) electron and truncated muon size spectra at sea level one can solve the EAS inverse problem - reconstruction of primary energy spectra and elemental composition, for not more than 2 kinds of primary nuclei.

## 1. Introduction

High accuracy of modern EAS experiments in the primary energy region of $10^{15}$ eV increased a number of publications on the solutions of the EAS inverse problem, which is reconstruction of primary nuclei energy spectra based on the detected EAS parameters at observation level [1,2,6,13]. However, the discrepancies in results continue to grow. In most of the cases, it is a result of the hidden experimental systematic errors due to uncertainties in the response functions of detectors. The uncertainty of A-A interaction model at these energies also contributes to the discrepancies in primary nuclei energy spectra results. However, there are certain publications [3,5,8,13] where the EAS inverse problem is solved based on erroneous presuppositions, which of course only increase the existing discrepancies of data.

## 2. EAS inverse problem

In general, the relation between energy spectra ($\partial\Im_A/\partial E$) of primary nuclei ($A \equiv 1, 4, \ldots 56$) and measured EAS electron and muon size spectra at observation level ($\Delta I/\Delta N^*_{e,\mu}$) at a given zenith angle $\theta$ is determined by an integral equation

$$\frac{\Delta I(\theta)}{\Delta N^*_{e,\mu}} = \sum_A \int\int \frac{\partial\Im_A}{\partial E}\frac{\partial W(E,A,\theta)}{\partial N_{e,\mu}}\frac{dG}{dN^*_{e,\mu}} dN_{e,\mu} dE \qquad (1)$$

where the EAS size spectra $\partial W/\partial N_{e,\mu}$ depend on observation level, $A, E, \theta$ parameters of primary nucleus and $A - A_{Air}$ interaction model, $dG(N,N^*)/dN^*$ is the error function of measurements.

Equation (1) is a typical ill-posed problem and has an infinite set of solutions for unknown primary energy spectra. However, the integral equation (1) turns to a

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Fredholm equation if a kind of a primary nucleus is defined directly in the experiment (as it successfully does in the balloon and satellite measurements where energy spectra for different nuclei are obtained up to $10^{15}$ eV).
As a result, the meaning of data presented in the [13] and further publications [3,5,8] where authors claim to have got the solution of equation (1) for 4 types of primary nuclei, is not clear.
Let's prove that using the measured EAS electron and truncated muon size spectra at 3 zenith angular intervals, as its done in [13], the Eq. (1) can have a single solution only for primary flux which consists of not more than 2 kinds of nuclei.
Let the EAS size spectra at observation level KASCADE ($t = 1020$ g/cm$^2$ [13]) be described by log-Gaussian distributions with mean $< N_{e,\mu}(E, A, \theta) >$ and variance $\sigma^2_{e,\mu}(E, A, \theta)$. It is known that this assumption is well performed at atmosphere depth $t > 700$ g/cm$^2$, primary energies $E > 10^5$ GeV and zenith angles $\theta < 35^0$.
Let also the measurements and further evaluations of EAS electron and truncated muon sizes be carried out without errors ($dG/dN^* \equiv \delta(N - N^*)$) and integral Eq. (1) include only statistical uncertainties.
Then a set of Eq. (1) transforms into the following:

$$\frac{\Delta I(\theta)}{\Delta N_{e,\mu}} = \sum_A \int \frac{\partial \Im_A}{\partial E} \frac{\partial W(E, A, \theta)}{\partial N_{e,\mu}} dE \tag{2}$$

Let's determine the parameters of distribution functions $\partial W/\partial N_{e,\mu}$ by the following known empirical expressions:

$$< N > \simeq a\Big(\frac{E}{1GeV}\Big)^b A^c \cos^d \theta, \quad \sigma \simeq \alpha A^\delta \Big(\frac{\ln(E/1GeV)}{\ln 10^6}\Big)^\varepsilon \cos^\rho \theta \tag{3}$$

where the values of corresponding approximation parameters $(a, \dots d, \alpha, \dots \rho)$ are presented in [12] and obtained by CORSIKA6016(NKG) EAS simulation code [4] at QGSJET interaction model [7]. The accuracy of approximation (3) for average EAS electron size $< N_e(E, A, \theta) >$ is less than 10% at $3 \cdot 10^5 < E < 3 \cdot 10^8$ GeV, $A \equiv 1, 4, \dots 56$, $\theta < 32^0$ and observation level 1020 g/cm$^2$. The corresponding accuracies of $\sigma_{e,\mu}$ and average EAS truncated muon size $N_\mu$ are less than 1-2%.
Changing the variables of kernel functions of Eq. (2) from $N_e$ and $N_\mu$ to $x_e$ and $x_\mu$ respectively according to $x_{e,\mu} \equiv A^\delta((N_{e,\mu}/ < N_{e,\mu} >) - 1) + 1$, we obtain the following set of Fredholm integral equations

$$\frac{\Delta I(\theta)}{\Delta N_{e,\mu}} = \int f_{e,\mu}(E, A) F_{e,\mu}(E, \theta) dE \tag{4}$$

where

$$f_{e,\mu}(E, A) = \sum_A \frac{\partial \Im_A}{\partial E} A^{\delta_{e,\mu} - c_{e,\mu}}, \quad F_{e,\mu}(E, \theta) = \frac{\partial W(E, \theta)}{\partial x_{e,\mu}} \frac{1}{a_{e,\mu} E^{b_{e,\mu}} \cos^{d_{e,\mu}} \theta} \tag{5}$$

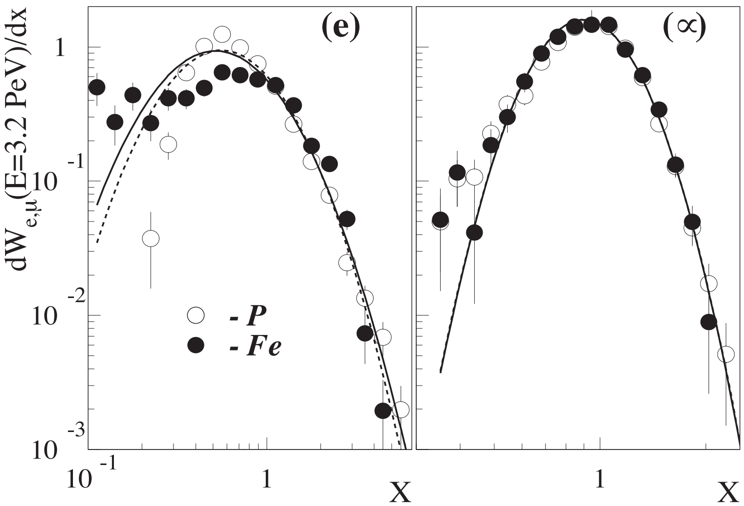


**Fig. 1.** EAS electron (left) and truncated muon (right) size spectra for primary Hydrogen and Iron nuclei at energy $E = 3.2$ PeV.

and kernel functions ($F_{e,\mu}$) independent of kind of primary nuclei.
Examples of distribution functions $\partial W/\partial x$ for EAS electron and truncated muon size spectra are presented Fig. 1. These data were obtained by CORSIKA code for primary proton ($A = 1$, empty symbols) and iron ($A = 56$, filled symbols) nuclei at energy $E = 3.2 \cdot 10^6$ GeV and zenith angle $\theta < 18^0$. The lines in Fig. 1 correspond to log-Gaussian distributions which were counted based on known values of mean $< x >= 1$ and variances $\sigma^2_{e,\mu}(E, \theta)$.
It is seen, that the distribution functions slightly depend on a kind of a primary nucleus, especially the right-hand sides of distributions. It is an important fact, because the contribution of the left-hand sides of distributions is negligible small at *a priori* steep primary energy spectra ($\partial\Im_A/\partial E \sim E^{-3}$).

Let the functions $f_e^{(j,k)}(E_i)$ and $f_\mu^{(j,k)}(E_i)$ be the solutions of the set of Fredholm equations (4) at $i = 1, \ldots m$ values of primary energies $E_i$, $j = 1, \ldots n$ kinds of primary nuclei and k=1,2,3 zenith angles. Then the values of unknown energy spectra ($\partial\Im_{A_j}/\partial E_i$) at a given set of $E_i$ and different primary nuclei ($A_j \equiv A_1, \ldots A_n$) are determined by the set of linear equations

$$< f_e^{(j)}(E_i) >_\theta = \sum_{A=A_1}^{A=A_n} A^{\delta_e - c_e} \frac{\partial\Im_A}{\partial E}\Big|_{E=E_i} \; : \; i = 1, \ldots m \tag{6}$$

$$< f_{\mu}^{(j)}(E_i) >_{\theta} = \sum_{A=A_1}^{A=A_n} A^{\delta_\mu - c_\mu} \frac{\partial \Im_A}{\partial E}\Big|_{E=E_i} \ : \ i = 1, \ldots m \qquad (7)$$

where $< f_{e,\mu}^{(j)}(E_i) >_{\theta} = \frac{1}{3}\sum_k f_{e,\mu}^{(j,k)}(E_i)$
Evidently, the single solutions of linear equations (6,7) occur only at $mn \leq 2m$, where $mn$ is a number of unknown $(\partial \Im_{A_j}/\partial E_i)$ and $2m$ is a number of equations. Therefore the condition of existence of solutions is $n \leq 2$ and the energy spectra [13] obtained based on KASCADE EAS electron and truncated muon size spectra for 4 ($A \equiv 1, 4, 16, 56$) primary nuclei have no physical meaning. At the same time the all-particle spectrum obtained in [13] have to be approximately right due to the power index in Eq. (7) $\delta_\mu - c_\mu \ll 1$ (see [12]).
It should be noted that unreliability of solutions [13] is also shown in [9] based on correlation analysis between the observable electron and truncated muon size spectra and unknown energy spectra of primary nuclei.

## 3. Conclusion

Based on equation (1), the investigation of the EAS inverse problem, which is the reconstruction of energy spectra of primary nuclei by the observable EAS electron and muon size spectra at observation level makes sense only using *a priori* given functions for unknown primary energy spectra with given unknown spectral parameters (so called parameterization of equation (1)) as it is done in [2,11].

## 4. References

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