Systematic Variation of Cosmic Ray Injection Across Supernova Shocks

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Abstract

The variation of the injection of suprathermal protons into the diffusive shock acceleration process at supernova remnant (SNR) shocks is considered. In the simplest case the shock can be approximated as being spherical in a uniform large-scale magnetic field. The injection rate depends strongly on the obliquity and diminishes as the angle between the ambient field and the shock normal increases. Therefore efficient particle injection takes place only in relatively small regions near the "poles," reducing the overall CR production. The sizes of these regions depend strongly on the random background field and the Alfvén wave turbulence generated due to the CR streaming instability. For the case of SN 1006 they correspond to about 20% of the entire shock surface. The results also explain the reduction of the effective injection rate by about two orders of magnitude relative to the value for a purely parallel shock.

1. Introduction

The time-dependent nonlinear kinetic theory of cosmic ray (CR) acceleration in supernova remnants (SNRs)[1], applied to the remnants of SN 1006, Tycho's supernova and Cas A [2-4], has demonstrated that the existing data are consistent with very efficient acceleration of CR nuclei at the SN shock wave, converting a significant fraction of the initial SNR energy content into CR energy.

At the same time an essential physical factor which strongly influences the final CR acceleration efficiency, is contained in our theory as a free parameter. This is the injection rate, described by a dimensionless parameter η that is a fixed small fraction of the interstellar medium (ISM) particles entering the shock front.

Unfortunately there is no complete selfconsistent theory of a collisionless shock transition, which can predict the value of the injection rate and its dependence on the shock parameters. For the case of a purely parallel shock hybrid simulations predict quite a high ion injection rate $\eta \sim 10^{-2}$ [5] that is consistent with analytical theory [6] and confirmed by measurements near the Earth's bow shock [7]. IHowever, in reality we deal with the evolution of the large-scale SN shock which expands into the ISM and its magnetic field. The leakage of

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suprathermal particles from the downstream region back upstream is is most efficient for a purely parallel subshock and becomes progressively less efficient when the shock is more and more oblique [6,8].

Here, we quantitatively consider the systematic variation of the injection rate across the SN shock surface in a simple approximation, taking the structure of the ambient magnetic field into account.

2. Results and discussion

The magnetic field structure in the downstream region is determined by the compression of the component perpendicular to the shock normal and is described by the relations $B_{2\parallel} = B_{1\parallel}$, $B_{2\perp} = \sigma B_{1\perp}$, where σ is the shock compression ratio, $B_\parallel = B\cos\theta$, $B_\perp = B\sin\theta$, θ is the angle between magnetic field and the shock normal, and the subscripts 1(2) correspond to the upstream (downstream) region.

The cold upstream plasma is advected with speed $u_1 = V_s$ towards the shock front, is compressed and heated, and flows with speed $u_2 = u_1/\sigma$ into the downstream region. Very fast particles from the thermalized downstream population, whose velocity component $v_\parallel$ exceeds some critical value v_{inj} and which move towards the shock front, are able to overtake it and to penetrate into the upstream region. Since these injected particles are supposed to be from the tail of a Maxwellian distribution we can write $\eta_\parallel = \eta(\theta_1 = 0) = \exp(-v_{inj}^2/v_{T2}^2)$, where v_{T2} is the mean thermal speed of the downstream particle population. Taking into account that $v_\parallel = v\cos\theta_2$ and $\cos^2\theta_2 = (1 + \sigma^2\tan^2\theta_1)^{-1}$ one finds $\eta(\theta_1) = \eta_\parallel^{1+\sigma^2\tan^2\theta_1}$. According to this relation the injection rate goes down quickly with increasing upstream angle θ_1 (Fig.1).

Efficient CR acceleration takes place when the injection rate exceeds a so-called critical injection rate [9]

$$\eta_{crit} = 10^{-1}(V_s/c)(p_{max}/mc)^{-1/4}, \tag{1}$$

where c is the speed of light and p_{max} is the maximum momentum of the accelerated CRs. For $V_s = 3000$ km/s and $p_{max} \sim 10^5\ mc$ the critical injection rate is $\eta_{crit} = 6 \times 10^{-5}$. This means that in the case of SN 1006, efficient CR production is expected to take place only within polar regions with $\theta_1 < \theta_{max} = 14°$. This angular width is considerably smaller than the one observed.

The existence of a random magnetic field component $\delta\vec{B}$ on scales large compared to the thickness of the subshock can change the value of the injection rate. To study this effect we assume that the ambient field $\vec{B}_1' = \vec{B}_1 + \delta\vec{B}$ consists of two components, the uniform field $\vec{B}_1$, and a superimposed, isotropically distributed random component $\delta\vec{B}$. If the spatial scale of the random component is much smaller then the shock size R_s, one can find the mean injection rate by averaging over the directions of the random field. The averaged injection rate

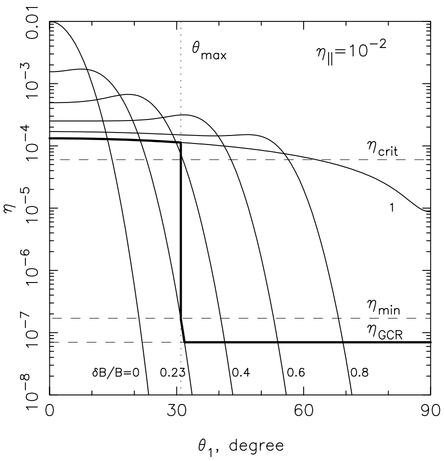

Fig. 1. The injection rate η as a function of the upstream angle θ_1 between the ambient magnetic field and the shock normal, for different amplitudes of the upstream random field $\delta B/B$ (*solid lines*). The *thick solid line* represents the injection rate expected for the current stage of SN 1006; the vertical *dotted line* indicates the angular region of efficient injection/acceleration in SN 1006.

as a function of angle θ_1 for different random field amplitudes $\delta B/B$ is shown in Fig.1.

The random magnetic field component $\delta\vec{B}$ can be created selfconsistently by the CR streaming instability in the upstream region [10]. The expected amplitude of the Alfvén waves excited due to the CR streaming instability is determined by the expression [11] $(\delta B/B)^2 = (V_s/c_a)P_c/(\rho V_s^2)$, where c_a is the Alfvén speed and P_c is the CR pressure at the shock front. With $v_{inj} \approx 2V_s$ [9] particles with $mc < p < p_{max}$ providing the main contribution to the CR pressure, we can write $(\delta B/B)^2 = (8c\eta/3c_a)\ln(p_{max}/mc)$. Once the selfconsistent Alfvén wave field δB exceeds the background ISM fluctuation field δB_0, any initially low injection rate leads to the growth of the random magnetic field in the upstream region which in turn leads to a nonlinear increase of the injection rate up to the level, which corresponds to $\delta B = B$, where B and δB are effective quntities. (see Fig.1). Equating $(\delta B/B)^2$ to the background level $(\delta B/B)_0^2$ we can find the minimal initial injection rate

$$\eta_{min} = 3c_a/(8c\ln(p_{max}/mc)(\delta B/B)_0^2. \tag{2}$$

The expected high efficiency injection region is bounded by the polar angle θ_{max} determined from the relation $\eta(\delta B, \theta_{max}) = \eta_{min}$, where $\eta(\delta B, \theta_1)$ is the function shown in Fig.1 for a given random field amplitude δB. Within the range $\theta_1 < \theta_{max}$, the expected selfconsistent injection rate corresponds to the curve $\eta(\delta B = B, \theta_1) \sim 10^{-4}$.

We apply the above formalism to the case of SN 1006, with $V_s = 3200$ km/s and $N_H = 0.1$ cm^{-3}. Since the interstellar random magnetic field is distributed over a wide range of scales $\lambda < 100$ pc, only part of this spectrum with scales $\lambda < \lambda_{max}$ has to be considered as a small scale field. λ_{max} can be taken as the diffusive length $l(p_{max})$ of the highest energy accelerated CRs. According to our numerical results [2], at the current evolutionary phase of SN 1006 $p_{max} = 4\times10^5 mc$ and $l(p_{max}) = 0.08R_s$. Assuming that the background ISM turbulent field is distributed according to the Kolmogorov law, we find that for scales smaller than $\lambda_{max} = 0.08R_s$ it has a value $(\delta B/B)_0^2 = /(B_0^2/8\pi) = 5.3 \times 10^{-2}$, or $(\delta B/B)_0 = 0.23$. For a typical ISM magnetic field $B_0 = 3\ \mu$G and a moderate maximum CR momentum $p_{max} = 10^3 mc$, we have $\eta_{min} = 1.7 \times 10^{-7}$.

The line $\eta(\theta_1)$ which corresponds to $\delta B/B = 0.23$ intersects the level $\eta_{min} = 1.7 \times 10^{-7}$ at $\theta_{max} = 31°$. Therefore the expected efficient injection rate $\eta \approx 10^{-4} > \eta_{crit}$ at $\theta_1 < 31°$ corresponds to the curve $\eta(\delta B = B, \theta_1)$. Then we have a situation where a sharp boundary $\theta_1 = \theta_{max}$ divides the regions of efficient and inefficient CR injection/acceleration. In reality this boundary is essentially smoothed by cross field diffusion. An approximate length of CR diffusion across the regular magnetic field in the upstream region is their diffusive length $l(p_{max})$. It corresponds to the angle interval $\Delta\theta_1 = (l/R_s)\mathrm{rad} \approx 5°$. Thus the smoothed region of efficient CR acceleration extends up to $\theta'_{max} = \theta_{max} + \Delta\theta \approx 36°$. Therefore efficient particle injection/acceleration is expected to take place within a bipolar region of about 20% of the shock surface. Its size corresponds rather well to that of the observed bright emission regions of SN 1006 [12].

We assume the spherically symmetric approach for the nonlinear particle acceleration process to be approximately valid in those shock regions where injection is efficient. To take the effective injection fraction $f_{\rm re} = 1 - \cos\theta'_{max}$ into account, we need then to introduce a *renormalization* factor for the nuclear CR acceleration efficiency and for all the effects which it produces in the SNR. According to the above estimate its value in the case of SN 1006 is $f_{\rm re} \approx 0.2$.

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