Transition Radiation from Radiators with Varying Periodicity

Gary L. Case and Michael L. Cherry
Dept. of Physics and Astronomy, Louisiana State Univ., Baton Rouge,LA 70803 USA

Abstract

X-ray transition radiation (TR) can be used to measure the Lorentz factors of relativistic particles. Typical transition radiation detectors (TRDs) use radiators with either regularly spaced foils or fibers or foam with irregular spacings. The TRD proposed for the ACCESS cosmic ray experiment employs radiators with a quasi-periodic structure, i.e., aluminum honeycomb. We have investigated the impact of quasi-periodic radiators on the expected TR yield, and present calculations comparing the TR yield of the quasi-periodic radiators to regularly spaced radiators.

1. Introduction

X-ray transition radiation produced by relativistic particles traversing periodic foil radiators has been studied in detail both theoretically [1-3] and experimentally [3-5]. In practice, irregular radiators (e.g., foams and fibers) have often been used with great success [6-9]. These can have the advantages of mechanical simplicity, ease of construction, and isotropic angular dependence. The total radiation intensity from such irregular radiators has been shown experimentally to be approximately the same as that from a periodic foil radiator with the same average foil thickness $\overline{l_1}$ and spacing $\overline{l_2}$ [10]. Recently, "quasi-periodic" radiators have been proposed with alternating short $(l_2 - \delta)$ and long $(l_2 + \delta)$ spacings, for example honeycomb and straw tube structures [11]. In the case of the proposed ACCESS cosmic ray experiment [11,12], an aluminum honeycomb radiator has been designed which possesses convenient mechanical properties and makes it possible to extend the range of energy measurements up to the range of Lorentz factors $\gamma \sim 10^5$. We investigate here the impact of such a regularly varying radiator spacing.

2. Theoretical Discussion

Previous studies have computed the radiation intensity from radiators with varying structures, for example for cases where the dielectric constant varies sinu-

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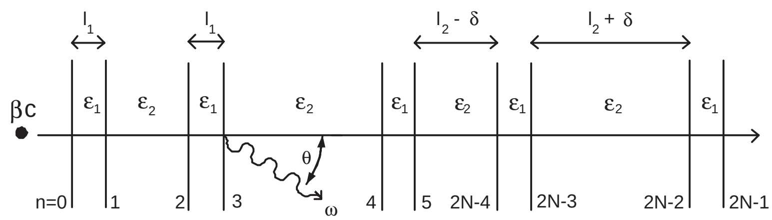

Fig. 1. Schematic of a charged particle traversing a radiator with alternating foil spacings.

soidally or randomly [3]. Here we consider the case where the foil thickness l_1 is constant and the spacing alternates between $l_2-\delta$ and $l_2+\delta$. In Fig. 1, a charged particle with constant velocity βc and Lorentz factor $\gamma \gg 1$ moves through a radiator consisting of N foils. Radiation is emitted at the $2N$ interfaces labelled from $n=0$ to $2N-1$. The differential intensity emitted per unit solid angle θ and frequency ω can be computed from the square of the sum of the complex amplitudes at each of the $2N$ interfaces [13]:

$$\frac{d^2S_N}{d\theta d\omega} = |\Psi(\theta)|^2 \sum_{j,k=0}^{2N-1} e^{i(\phi_j-\phi_k^*)} \quad , \tag{1}$$

where $\Psi(\theta)$ is the radiation amplitude from a single interface and ϕ_j is the phase factor corresponding to interface j. Since the radiator period is $2l_1+2l_2$, we express the phase factors in terms of $\Delta = 4l_1/Z_1 + 4l_2/Z_2$, where Z_i are the standard formation zones

$$Z_i = \frac{4\beta c}{\omega}\left(\frac{1}{\gamma^2}+\theta^2+\frac{\omega_i^2}{\omega^2}\right)^{-1} \tag{2}$$

in the foils ($i=1$) and the gas/vacuum gaps ($i=2$). Here ω_i is the plasma frequency in medium i, related to the dielectric constant at x-ray energies by $\epsilon_i = 1-\omega_i^2/\omega^2$. The phase factors appropriate to each interface can then be written in the form

$$\begin{array}{cc} \phi_{4j} = j\Delta & \phi_{4j+1} = j\Delta + \frac{2l_1}{Z_1} \\ \phi_{4j+2} = j\Delta + \frac{2l_1}{Z_1} + \frac{2(l_2-\delta)}{Z_2} & \phi_{4j+3} = j\Delta + \frac{4l_1}{Z_1} + \frac{2(l_2-\delta)}{Z_2} \end{array} \tag{3}$$

where j ranges from 0 to $N/2-1$. The sum in Eq. (1) can then be expressed as

$$S = \sum_{j,k=0}^{2N-1} e^{i(\phi_j-\phi_k^*)} = S_o A A^* \tag{4}$$

where

$$S_o = \sum_{j=0}^{N/2-1} e^{i\phi_{4j}} \sum_{k=0}^{N/2-1} e^{-i\phi_{4k}^*} \quad \text{and} \quad A = \left[1+e^{2i(\frac{l_1}{Z_1}+\frac{l_2-\delta}{Z_2})}\right]\left[1+e^{\frac{2il_1}{Z_1}}\right] \quad . \tag{5}$$

In the case where absorption and scattering are not important (i.e., where Z_i is real), then

$$AA^* = 4\left[1+\cos\left(\frac{2l_1}{Z_1}\right)\right]\left[1+\cos\left(\frac{2l_1}{Z_1}+\frac{2(l_2-\delta)}{Z_2}\right)\right] \qquad (6)$$

and one can define the ratio of the TR yield in the quasi-periodic radiator to the yield from the regularly spaced radiator,

$$R = \frac{AA^*(\delta)}{AA^*(\delta=0)} = \frac{1+\cos\left(\frac{2l_1}{Z_1}+\frac{2l_2}{Z_2}-\frac{2\delta}{Z_2}\right)}{1+\cos\left(\frac{2l_1}{Z_1}+\frac{2l_2}{Z_2}\right)} \quad . \qquad (7)$$

Fig. 2 shows an example of the aluminum honeycomb radiator structure recently proposed for the ACCESS mission [11]. Particles passing vertically downward through the radiator encounter one of two radiator structures: Particles with type 1 trajectories encounter a regular structure with $l_1 = 150\,\mu$m and $l_2 = 5.2$ mm; and particles with type 2 trajectories encounter a quasi-periodic radiator structure with foil thickness $l_1 = 75\,\mu\text{m}/\sin 41^\circ = 120\,\mu$m and spacing alternating between $l_2 = 2.6\,\text{mm}-\delta$ and $2.6\,\text{mm}+\delta$, where δ is constant for each (vertical) trajectory but varies over the range $0 < \delta < 2.6$ mm depending on the lateral position of the trajectory.

The effect of the quasi-periodic structure can be determined by evaluating the average R:

$$\overline{R} = \frac{1}{2\delta_{\max}}\int_{-\delta_{\max}}^{\delta_{\max}} R\,d\delta = \frac{1+\frac{Z_2}{2\delta_{\max}}\cos\left(\frac{2l_1}{Z_1}+\frac{2l_2}{Z_2}\right)\sin\frac{2\delta_{\max}}{Z_2}}{1+\cos\left(\frac{2l_1}{Z_1}+\frac{2l_2}{Z_2}\right)} \quad . \qquad (8)$$

3. Conclusion

For $\delta_{\max} \ll Z_2$, $\overline{R} \approx 1$. This is the case for high Lorentz factors, high x-ray energies and small angles (see Eq. 2). However, as $\delta_{\max}$ approaches $Z_2/2$, $\overline{R}$

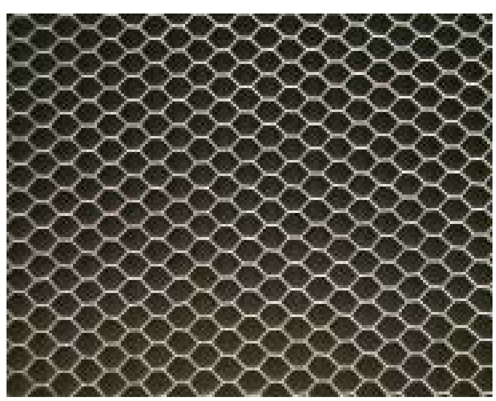

Fig. 2. Aluminum honeycomb structure

begins to oscillate on either side of unity. The effect of a finite $\delta_{\max}$ is to enable resonances (depending on the radiator geometry and the values of Z_1 and Z_2) where $\overline{R} \gg 1$, indicating an enhancement in the TR signal relative to the regular spaced foils. Detectors sensitive to the x-ray energies where these enhancements occur (in particular where $\cos(2l_1/Z_1+2l_2/Z_2) \approx -1$) can then be used to increase the TRD performance over regularly spaced foil TRDs.

4. References

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