Simulating Particle Acceleration in Modified Shocks Using a New Coarse-grained Finite Momentum-Volume Scheme

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Abstract

We are developing a new, fast numerical scheme for the CR diffusion convection equation to be applied to the study of the nonlinear time evolution of CR modified shocks for arbitrary spatial diffusion properties. The scheme uses a coarse-grained finite volume method in momentum space that is an extension of methods introduced by us previously. This approach has enabled us to carry out simulations using only 6-10 momentum bins spanning several orders of magnitude in momentum that agree well in comparisons with results from simulations of modified shocks carried out with our conventional finite difference scheme based on more than an order of magnitude more momentum points. The coarse-grained solutions are faster by a factor that scales approximately with the reduction in the number of momentum bins.

1. Introduction

To reduce the effort required to solve the diffusion convection equation (DCE) during diffusive shock acceleration (DSA), we take advantage of the expected smooth structure of the particle distribution function, $f(p)$. We use a finite volume approach in momentum space along with a simple model for the distribution inside individual volumes of momentum space. The method extends that in [1] and [2] and is similar to the scheme introduced by [3], but computationally simpler. Those authors actually ignored spatial diffusion, so could not explicitly treat DSA. They applied analytic test-particle DCE solutions for $f(p)$ at shock jumps.

2. The Method

Ignoring momentum diffusion for now we can write in 1D

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\kappa\frac{\partial f}{\partial x}\right) + \frac{1}{3}\left(\frac{\partial u}{\partial x}\right)p\frac{\partial f}{\partial p} + Q, \tag{1}$$

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where S is a convenient source term, and all the other symbols take their usual meanings.

The number of CRs in the momentum bin $\Delta p_i = [p_i, p_{i+1}]$ is

$$n_i = \int_{p_i}^{p_{i+1}} p^2 f(p) dp. \tag{2}$$

Integrating over the finite momentum volume bounded by Δp_i gives

$$\frac{\partial n_i}{\partial t} + \frac{\partial u n_i}{\partial x} = F_{n_i} - F_{n_{i+1}} + \frac{\partial}{\partial x}\left(K_{n_i}\frac{\partial n_i}{\partial x}\right) + S_{n_i}, \tag{3}$$

where $F_{n_i} = \dot{p}_i p_i^2 f(p_i)$, with the "momentum speed", $\dot{p} = -p\frac{1}{3}\frac{\partial u}{\partial x}$,

$$K_{n_i} = \frac{\int_{p_i}^{p_{i+1}} \kappa \frac{\partial f}{\partial x} p^2 dp}{\int_{p_i}^{p_{i+1}} \frac{\partial f}{\partial x} p^2 dp} \Rightarrow \frac{\int_{p_i}^{p_{i+1}} \kappa f p^2 dp}{n_i}, \tag{4}$$

and

$$S_{n_i} = \int_{p_i}^{p_{i+1}} p^2 S(p) dp. \tag{5}$$

The first pair of terms on the RHS of equation (3) represents the net flux of CRs across the boundaries of the individual momentum bins due to adiabatic flow compression or expansion. Extension of this term to include fluxes due to other energy loss or gain processes, such as momentum diffusion or radiative losses is obvious.

In addition we define

$$g_i = \int_{p_i}^{p_{i+1}} p^3 f(p) dp. \tag{6}$$

For relativistic particles g_i is proportional to the CR energy in the bin.

This moment of the DCE is

$$\frac{\partial g_i}{\partial t} + \frac{\partial u g_i}{\partial x} = \frac{1}{3}\left(\frac{\partial u}{\partial x}\right)\left[(p^4 f)|_{p_i}^{p_{i+1}} - g_i\right] + \frac{\partial}{\partial x}\left(K_{g_i}\frac{\partial g_i}{\partial x}\right) + S_{g_i}, \tag{7}$$

where

$$K_{g_i} = \frac{\int_{p_i}^{p_{i+1}} \kappa \frac{\partial f}{\partial x} p^g dp}{\int_{p_i}^{p_{i+1}} \frac{\partial f}{\partial x} p^3 dp} \Rightarrow \frac{\int_{p_i}^{p_{i+1}} \kappa f p^3 dp}{g_i}. \tag{8}$$

Since the local slope, $\frac{\partial \ln f}{\partial \ln p} = -q(p)$, is a slowly varying function over much of momentum space, the simplest natural subgrid model for $f(p)$ assumes q is constant inside Δp_i; that is, we can apply a piecewise powerlaw model for $f(p)$ and use a logarithmic spacing in the momentum grid. Then, for example,

$$n_i = \frac{f_i p_i^3}{q_i - 3}\left[1 - d_i^{3-q_i}\right], \tag{9}$$

with obvious extension to g_i and the other terms in the DCE moment equations, where $f_i = f(p_i) = (p_{i+1}/p_i)^{q_i} f_{i+1}$, and $d_i = p_{i+1}/p_i$. Using the ratio $g_i/p_i n_i$ one can derive the intrabin index, q_i once n_i and g_i are known.

To update the distribution function we integrate equations (3) and (7) over the timestep Δt^k and operator split into three parts. Spatial advection is carried out by an explicit van Leer scheme and spatial diffusion by a semi-implicit Crank Nicholson scheme. Momentum advection, involving terms like F_{n_i} is carried out with an explicit scheme illustrated for n_i by

$$n_i^{k+1} = n_i^k + \Delta t^k * (\overline{F_{n_i}} - \overline{F_{n_i}}), \tag{10}$$

where the overbar indicates a flux averaged over the domain of dependence in momentum over the timestep.

3. Discussion

Fig 1. shows results from an initial test of the scheme inside a TVD hydrodynamics code and, for comparison, the equivalent simulation with a conventional finite difference scheme for solving the DCE [4]. The simulations involve evolution of an initial Mach 10 shock discontinuity with an initially uniform CR pressure equal to twice the upstream gas pressure with $f(p) \propto p^{-5}$. The diffusion coefficient was $\kappa(p) = p^{0.5}$. In diffusion time units, $t_d = \kappa(p=1)/u_s^2$, the shock structure and immediate-postshock particle distribution function are shown at t = 0, 2.5, 5, 7.5, 10. The conventional DCE solution used 96 momentum points spanning the momentum range ln(p) = [-1, 8]. The coarse-grained solution used 9 logarithmically expanding momentum bins. It gave a factor 5 reduction in computation time. The code has been tested with simple CR injection models and we are presently implementing our thermal injection scheme [4] and porting the routine to our CR AMR code, CRASH.

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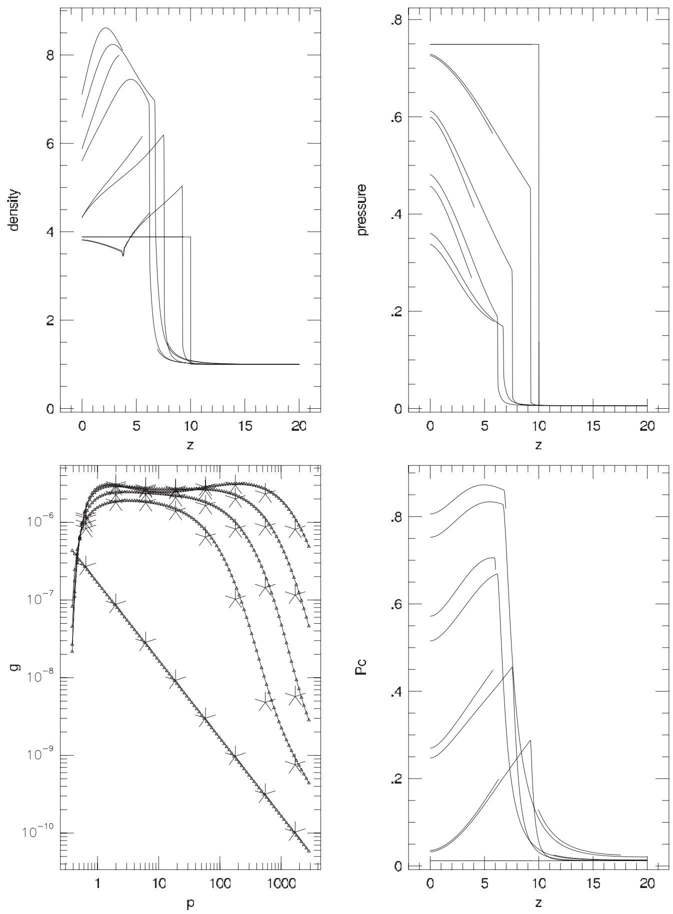

Fig. 1. Evolution of a CR modified Mach 10 plane shock using two different schemes for solving the diffusion convection equation. The solid lines (dynamical variables) and the large stars (distribution function) represent solutions based on 8 momentum bins and the new scheme described in the text. The dotted lines and small triangles come from a conventional finite difference scheme using 96 momentum points. The new code took about 80% less execution time.