
Electron And Proton Acceleration In SNR

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Abstract

In this work we discuss the theoretically predicted spectra of electrons and protons inside young Supernova Remnants. Assuming a rigidity dependent acceleration mechanism, the relation between the electron and proton spectra is determined by their different injection and energy losses properties. The importance of gradients in the spatial distributions of the accelerated particle density and the magnetic field is stressed.

1. Introduction

Supernova Remnants (SNRs) are the main candidate as the site for cosmic ray acceleration [3]. Recently robust evidence has emerged that some young SNRs contain a large population of accelerated electrons [5, 7, 1] while the evidence for the presence of relativistic hadrons remain uncertain [4, 2, 6]. In this work we want to discuss the theoretically expected form of the spectra of relativistic particles inside a SNR. It is well known that the 1st order Fermi mechanism predicts that the source spectrum of particles injected in the galaxy is a power law $Q(E) \propto E^{-\alpha_s}$ with a source exponent $\alpha_s \simeq 2$, up to a maximum energy that for protons is determined by the failure of the acceleration mechanism, and for electrons is determined by the competition of acceleration with the synchrotron energy losses. This result is in good agreement with the measurements of the p and e^- spectra, that can be fitted (at sufficiently high energy) with power laws with exponents $\alpha_p \simeq 2.7$ and $\alpha_e \simeq 3.1$. Taking into account the dominant loss mechanisms, that are synchrotron and Inverse Compton (IC) scattering for electrons (with $-dE/dt \propto E_e^2$), and escape from the galaxy for protons (with confinement time $\tau_p \propto E^{-0.6}$), this is roughly consistent with $Q_e \sim Q_p \sim E^{-2.1}$, with the significant puzzle that the normalizations of the e and p source spectra differ by nearly two order of magnitude, pointing to a significant difference in the accelerator injection efficiency. To study the radiation from SNRs it is however important to discuss in detail the form of the spectrum *inside* the accelerator, and the highest achievable electron energy clearly plays a key role. In this work we will argue that the spectra of relativistic particles in a young SNR of age t should be considered as the sum of two components, a first one of particles that are still being accelerated, and a second one of particles left behind by the SN

shock that remain “stored” inside the SNR. For electrons one can write:

$$N_{e,p}^{\text{SNR}} = N_{e,p}^{\text{acc}} + N_{e,p}^{\text{storage}} \quad (1)$$

The two populations should have different spatial distribution with the particles in the acceleration stage forming a shell around the propagating spherical shock wave, and the stored particles filling the inside volume.

2. Relativistic particles in Supernova Remnants

The evolution with time of the particles that are accelerated can be approximately described with the equation:

$$\frac{\partial N_{e,p}^{\text{acc}}(E, t)}{\partial t} = S_{e,p}(E, t) - \frac{\partial}{\partial E} \left\{ [\beta_{\text{acc}}(E) + \beta_{\text{loss}}^{e,p}(E)] N_{e,p}^{\text{acc}}(E, t) \right\} - \frac{N_{e,p}^{\text{acc}}(E, t)}{\tau_{\text{esc}}(E)} \quad (2)$$

The first term on the right side of (2) represents injection, that for simplicity can be modeled as: $S(E, t) \simeq S_0(t) \delta[E - E_0]$. The second term in (2) takes into account acceleration and energy losses. The acceleration term can be written as:

$$\beta_{\text{acc}}(E) = \xi E / T_{\text{cycle}}(E) \simeq \xi E / (\tau E) \simeq \xi / \tau \quad (3)$$

where $T_{\text{cycle}}(E)$ is the “cycle time” for a particle of initial energy E to pass twice across the shock, and $\Delta E \simeq \xi E$ is the average energy acquired during a cycle (with $\xi = 4/3(\beta_1 - \beta_2)$). The cycle time can be written as: $T_{\text{cycle}} = 4 [(D_1/u_1) + (D_2/u_2)]$ where $u_{1,2}$ are the velocity of the fluid on the upstream and downstream sides of the shock and $D_{1,2}$ the diffusion coefficients, that depend on the structure of the turbulence in the magnetic field. Making the assumption that the diffusion coefficients are proportional to the Larmor radius of the particles $D \simeq (r_L c)/3 \simeq Ec/(3qB)$ it follows that T_{cycle} is linear in E and the acceleration rate is constant. The energy loss term is important only for electrons, and is dominated by the synchrotron emission with a smaller contribution of IC on the cosmic microwave background radiation (CMBR). Both contributions are quadratic in energy: $\beta_{\text{loss}} \simeq -b^2 E^2$ (with $b \propto B^2$). The last term in (2) describes the escape of particles from the acceleration region. The probability P_{esc} that during one cycle a particle “exits” the accelerator, remaining in the downstream region, is approximately constant, and can be estimated as $P_{\text{esc}} \simeq 4/3(\beta_1 - \beta_2) \simeq \xi$. The escape time is then $\tau_{\text{esc}} = T_{\text{cycle}}/P_{\text{esc}}$.

For a constant injection rate, and neglecting the energy loss term (as it is a good approximation for protons) one can easily find the stationary solution: $N_p^{\text{acc}}(E) \propto E^{-\alpha} T_{\text{cycle}}(E)$ with $\alpha = 1 + P_{\text{esc}}/\xi \simeq 2$. This corresponds to the well known result that the injection of cosmic rays $Q(E) = N^{\text{acc}} \times P_{\text{esc}}/T_{\text{cycle}}(E)$ is a power law with a universal exponent $\alpha \simeq 2$ (this result is also independent from the form of T_{cycle}). Note that the spectrum of particles inside the accelerator is harder than the spectrum injected in the galaxy.

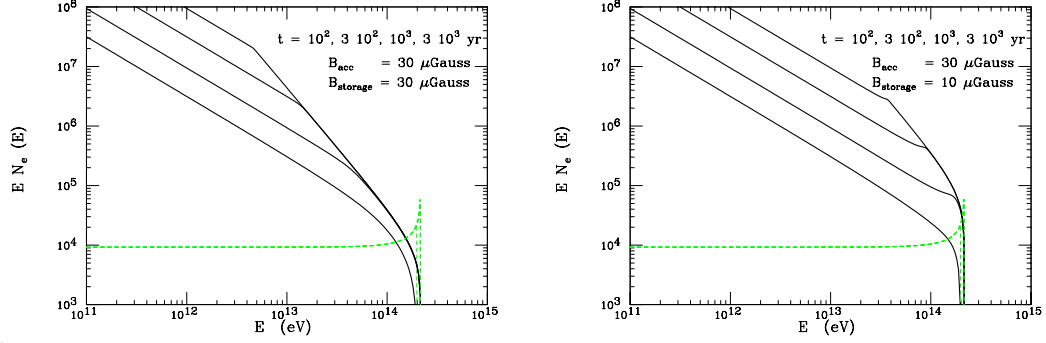


Fig. 1. Relativistic electron population in the “accelerator” (dashed lines) and “storage” volumes (solid lines) of a SNR of age $t(\text{years}) = 10^2, 3 \times 10^2, 10^3$ and 3×10^3 . The calculation is performed assuming a constant injection rate. In the left (right) panel the magnetic field is chosen as $B = 30 \mu\text{Gauss}$ in the acceleration region, and 30 (10) μGauss in the storage region.

Including the energy loss there will be a maximum possible energy determined by condition: $\beta_{\text{acc}} + \beta_{\text{loss}} = 0$. Using the form $T_{\text{cycle}}(E) = \tau E$: $E_{\text{max}} \simeq \sqrt{\xi/(\tau b)}$. For a constant injection the stationary solution becomes:

$$N_e^{\text{acc}}(E) \propto (E/E_{\text{max}})^{-\alpha} \left[1 - (E/E_{\text{max}})^2\right]^{-1+\frac{\alpha}{2}} \quad (4)$$

It is simple to compute the energy spectrum of the particles that exit from the accelerator and remain stored inside the SN shell as:

$$N_e^{\text{storage}}(E) = \int_0^t dt' \int dE_0 Q_e(E_0, t) \times P[E; E_0, t - t'] \quad (5)$$

where $Q_e(E_0, t) = N_e^{\text{acc}}/\tau_{\text{esc}}(E)$ is the injection rate in the storage volume, and $P[E, E_0, t]$ is the probability that an electron injected with energy E_0 remains with energy E after a time t , taking into account the energy losses, (dominated by the synchrotron emission and controlled by the value of the B in the storage region). As an illustration in fig. 1 we show the energy spectrum of electrons in the accelerator and storage region of an hypothetical SNR, at different times, calculated with the assumption that the injection in the accelerator is constant. Clearly the electron populations in the accelerator region are identical in the two calculations, however in the case where the magnetic field is weaker in the storage volume, N_E^{storage} is larger, because of the effect of the weaker energy loss.

It is straightforward to compute the radiation from the calculated e populations. The two most important contributions are synchrotron emission, and IC scattering on the CMBR. The results (for a time $t = 1000$ years) are shown in fig. 3. A comparison of the results obtained assuming different values of the magnetic field in the storage region clearly shows that a smaller B_{storage} results in two effects: (i) a weaker synchrotron emission, and (ii) a stronger inverse Compton contribution in the high energy region. The first effect is the obvious consequence

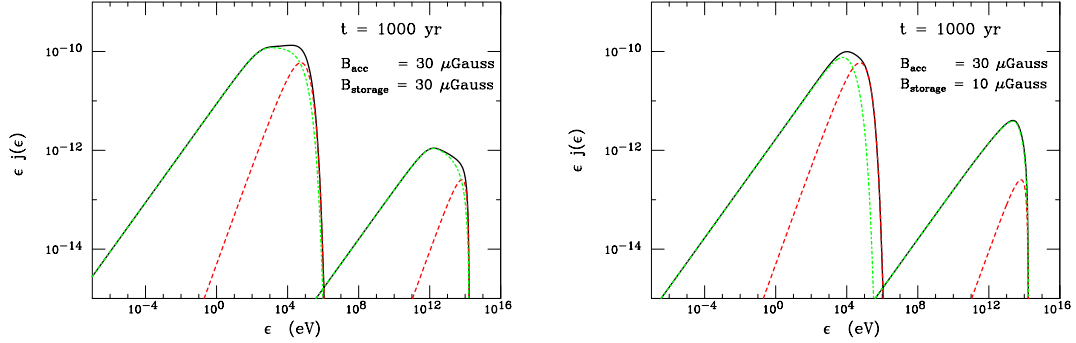


Fig. 2. Emissivity for synchrotron radiation and Inverse Compton scattering on the 2.7K radiation averaged over the entire volume of a SNR of age $t = 10^3$. The calculation is performed assuming a constant injection rate. In the left (right) panel the magnetic field is assumed to be $B = 30 \mu\text{Gauss}$ in the acceleration region, and 30 (10) μGauss in the “storage region” at the center of the shell. The contributions of electrons in the acceleration (storage) region is shown with dashed (dot–dashed) lines.

of the fact that synchrotron emission is $\propto B^2$; on the other hand, a weaker magnetic field results in a larger high energy population of stored electrons, this is not sufficient to offset the effect of a smaller B for synchrotron radiation, however it clearly implies a larger IC emission.

In [7] the multiwavelength spectrum of SN1006 has been fitted as the combination of a synchrotron and IC components (on the CMBR) of a population of relativistic electrons with spectrum $N_e \propto E^{-2.2} \times \exp[-E/(51 \text{ TeV})]$ filling a volume with a uniform magnetic field: $B = 4 \pm 1 \mu\text{Gauss}$. This value seems surprisingly small because it is similar to the galactic field, and is significantly smaller than the equipartition field. Introducing a gradient for the magnetic field allows to obtain similar power in the synchrotron and IC components having a much stronger field in the acceleration region closer to the shock. This should also be reflected in a different spatial distribution of the sources of radiation in the X–ray and TeV energy ranges. Directions for further studies are the introduction of a gradient in the magnetic field and a more realistic time dependence of the injection rate in the acceleration process.

References

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