Monte Carlo Simulations of Electron Acceleration in Parallel Relativistic Shocks

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Abstract

We have used Monte Carlo simulations to study the properties of particle acceleration in relativistic shocks that have a non-trivial structure: a finite width and a specific velocity profile. The numerical modeling indicates that (i) rigidity dependence of the mean free path and (ii) the electron injection energy crucially affect the shape of the electron spectrum. For an energy-independent mean free path, a spectral index of 3.2 of accelerated electrons is obtained for ultra-relativistic shocks with a thickness determined by ion dynamics. The value of 2.2 previously computed by several authors for a step-like shock is obtained as a high-energy limit for a mean free path increasing as a function of energy.

1. Introduction

Relativistic bulk motion (toward the observer) of ultra-relativistic electron populations radiating via the synchrotron and the inverse Compton mechanisms provides the most widely accepted model to account for the rapidly varying electromagnetic emission extending to gamma-ray frequencies from blazar-type active galaxies [3] and gamma-ray bursts [2]. In situations involving relativistic bulk motions and accelerated electrons, it is rather natural to assume that the latter result from particle acceleration in relativistic shocks, presumably by the first-order Fermi mechanism. The mechanism is relatively well understood if the shock wave can be approximated as a discontinuity propagating parallel to the mean ambient magnetic field [1]. If the shock has non-trivial internal structure, however, its acceleration efficiency is more poorly understood (see, however, [4]). In this paper, we will study electron acceleration in parallel shocks with finite thickness.

2. Model

We have performed test-particle simulations of electron acceleration in shock waves with finite width. The simulations trace individual electrons under the guiding-center approximation in a homogeneous background magnetic field

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with superposed (magnetic) scattering centers frozen-in to the plasma flow. Scatterings off the irregularities are simulated making small random displacements of the tip of the electron's momentum vector using a random generator (see [5, 7]).

The mean free path, λ, of all charged particles is taken to be a power-law function of particle rigidity, consistent with the assumed magnetic nature of scattering. We consider relativistic particles ($\gamma \gg 1$), with speeds close to that of light. Such particles are efficiently scattered by Alfvén waves, and these wave–particle interactions can be, to the lowest approximation, described by quasi-linear theory. Thus, the scattering frequency of relativistic particles, $\nu = c/\lambda$, of species i is

$$\nu_i(\gamma') \approx \nu_0 \left(\frac{m_e \Gamma_1}{m_i \gamma'} \right)^{2-q}, \tag{1}$$

where ν_0 and q are parameters depending on the spectrum of magnetic fluctuations. The scatterings are performed in the local rest frame, denoted by primes, so the Lorentz factor is also measured in that frame. Note that we have simplified the numerical treatment by neglecting the dependence of ν on pitch angle.

In standard quasilinear theory, q is the spectral index of the magnetic fluctuations causing the scattering. Two values for this parameter are considered: (Q1) $q = 2$ giving an energy independent mean free path; and (Q2) $q = 5/3$ corresponding to the Kolmogorov spectrum of turbulence. Proton rigidity at constant Lorentz factor is m_p/m_e times the electron rigidity. Thus, the proton mean free path is $(m_p/m_e)^{2-q}$ times the electron mean free path. Thus, a shock wave having a thickness of about one thermal-proton mean free path, $\sim \lambda_{p,\,\mathrm{th}} = c/\nu_p(\Gamma_1)$, seems like a thick structure to all electrons with Lorentz factors less than $\gamma' \sim \Gamma_1 (m_p/m_e)^{2-q}$, and electron acceleration at these energies should be modest, resembling adiabatic compression.

We have studied two velocity profiles: (U1) the tanh profile of Schneider & Kirk [4]

$$u(x) = \frac{u_1 + u_2}{2} - \frac{u_1 - u_2}{2} \tanh \frac{x}{W}, \tag{U1}$$

where W is the width of the shock and x axis points in the direction of the flow in the shock frame; and (U2) a modified profile of

$$u(x) = u_1 - (u_1 - u_2) \mathcal{H}(x) \tanh \frac{2.4\, x}{\lambda_{p,\,\mathrm{th}}}, \tag{U2}$$

obtained by fitting the results of a self-consistent Monte-Carlo simulation of shock structure described elsewhere [6]. Here $\mathcal{H}(x)$ is the step function. To make the two models comparable, the shock width W has to be adjusted so that the transition from upstream (u_1) to downstream (u_2) values takes place over the same distance in both models. For this, we use the width of the region where $u(x)$ is in the range $u_1 - \delta u < u < u_2 + \delta u$ with $\delta u = 0.01\, c$. This gives $W = \lambda_{p,\,\mathrm{th}}/4.2$. We use the value of $\Gamma_1 = 10$ for the upstream bulk Lorentz factor, and $u_1 u_2 = \frac{1}{3} c^2$.

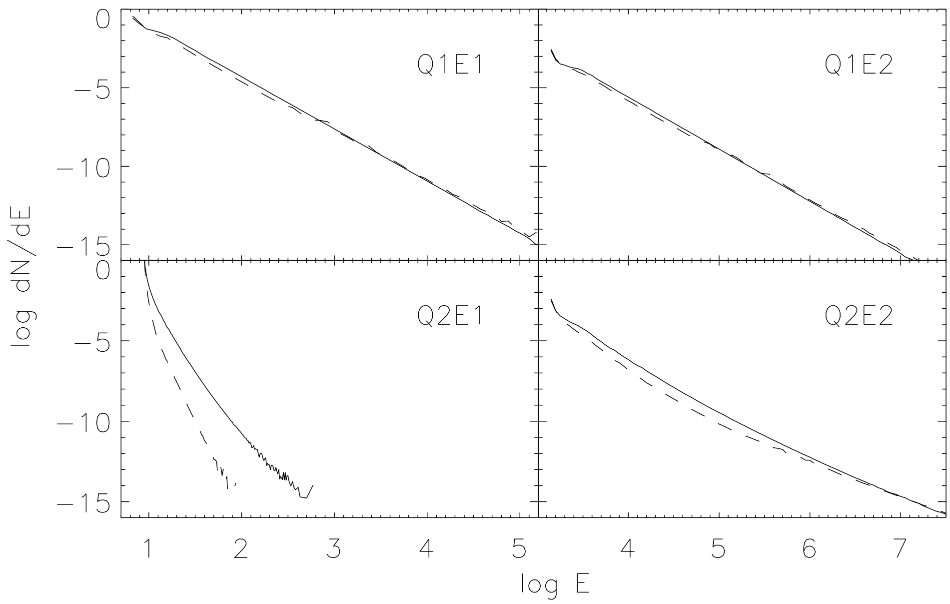

Fig. 1. The energy spectra of accelerated electrons in shocks with finite thickness. Solid [dashed] curves correspond to speed profile (U1) [(U2)]. See text for a description of the different models.

Electrons are injected to the acceleration process in the downstream region. We consider two models for the injection energy: (E1) a "kinematic" injection energy $\gamma = \Gamma_\Delta = \Gamma_2\Gamma_1(1 - u_1u_2/c^2)$, i.e., the energy of cold upstream electrons as seen from the downstream gas, and (E2) a "thermalized" injection energy $\gamma = \frac{1}{2}\alpha\Gamma_\Delta(m_p/m_e)$, i.e., a fraction of proton thermal energy in the downstream region ($\alpha = 1$ corresponding to equipartition). We use $\alpha = 0.2$ throughout this work.

3. Results and Discussion

The results of the simulated electron spectrum for all eight models (Q1U1E1, Q2U1E1, ..., Q2U2E2) are plotted in Figure 1. The results show that the two velocity profiles produce different results, in the case Q2: for the kinematic injection (E1), the speed profile (U1) produces a significantly harder spectrum than the speed profile (U2), and the results are slightly different even for the thermal injection model. The reason for the differences is probably that (U1) has a larger maximum value of the speed gradient than (U2). The fact that the results are so similar in the case Q2, however, indicates that the adjustment of the shock width for U1 was reasonable.

Differences between the two turbulence models are significant. The spectrum in the case Q1 is a power law with a spectral index of ~ 3.2 independent of the injection energy, as expected. The spectral shape in the case Q2 is not a power law but hardens as a function of energy, because for a mean free path increasing with energy, the shock seems thinner for electrons at higher energies. In the case E1, the Kolmogorov scattering law (Q2) produces accelerated particles much less efficiently than the case of energy-independent mean free path. The thermalized injection yields accelerated particle populations in both turbulence models. At the highest energies, the spectral index in the Q2E2 model approaches the value of 2.2 obtained for a step-like shock as $\Gamma_1 \gg 1$ [1].

In conclusion, electron acceleration in parallel relativistic shock waves with non-trivial internal structure is heavily depending on the rigidity dependence of the particle mean free path. For a shock thickness determined by ion dynamics and a mean free path increasing with energy, the standard power-law electron spectra can be obtained only at very high energies, e.g., at $\gamma > 10^5$ for $\lambda \propto \gamma^{1/3}$.

4. Nomenclature

c	speed of light $= 3 \cdot 10^{10}$ cm s^{-1}	q	Spectral index of magnetic fluctuations
x	Particle position, cm	W	Shock width, cm
m_i	Particle mass, g	γ	Particle Lorentz factor
ν_i	Scattering frequency, s^{-1}	λ_i	Mean free path, cm
u	Bulk flow speed, cm s^{-1}	Γ	Bulk-speed Lorentz factor
α	Parameter determining the electron injection energy		
Γ_Δ	Upstream bulk-Lorentz factor as seen from the downstream rest frame		

The symbol γ' refers to values measured in the local rest frame, and γ to values measured in the shock frame. Subscripts 1 and 2 in u and Γ refer to quantities measured far upstream and far downstream the shock wave, respectively. Subscript i above numbers the particle species with e and p referring to electrons and protons. Subscript 'th' refers to downstream thermal values.

5. References

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