

---

# First-order Fermi Particle Acceleration at Relativistic Shock Waves with a ‘Realistic’ Magnetic Field Turbulence Model

---

Jacek Niemiec,<sup>1</sup> and Michał Ostrowski<sup>2</sup>

(1) *Institute of Nuclear Physics, ul. Radzikowskiego 152, 31-342 Kraków, Poland*

(2) *Astronomical Observatory, Jagiellonian University, ul. Orla 171, 30-244 Kraków, Poland*

---

## Abstract

First-order Fermi acceleration process at a relativistic shock wave is investigated by means of Monte Carlo simulations involving numerical integration of particle equations of motion in a turbulent magnetic field near the shock. In comparison to earlier studies a few ‘realistic’ features of the magnetic field structure are included. The upstream field consists of a mean field component inclined at some angle to the shock normal and finite-amplitude perturbations imposed upon it. The perturbations are assumed to be static in the local plasma rest frame. We apply an analytic model for the turbulence with a flat or a Kolmogorov spectrum within a finite (wide) wave vector range. The magnetic field is continuous across the shock — the downstream field structure is derived from the upstream one from the hydrodynamical shock jump conditions. We present and discuss the obtained particle spectra and angular distributions at mildly relativistic sub- and superluminal shocks. We show that particle spectra diverge from a simple power-law, an exact shape of the spectrum depends on both an amplitude of the magnetic field perturbations and the considered wave power spectrum.

## 1. Introduction

At a relativistic shock wave the bulk velocity of the flow is comparable to particle velocity. This leads to anisotropy of particle angular distribution which can substantially influence the process of particle acceleration. In contrast to the nonrelativistic case, the particle power-law spectral indices depend on the conditions at the shock, including the spectrum and amplitude of magnetic field perturbations and the mean field inclination to the shock normal [1-3, 5-10].

In the case of weakly perturbed magnetic field the acceleration process can be investigated via analytical methods [5-7]. However, if finite-amplitude MHD waves are present in the medium these approaches are no longer valid and numerical methods have to be used. The particle acceleration studies so far applied very simple models for numerical modeling of the perturbed magnetic field

structure [1-3, 9, 10]. The purpose of the present work is to simulate the first-order Fermi acceleration process at mildly relativistic shock waves propagating in more realistic perturbed magnetic fields, taking into account a wide wave vector range turbulence with the power-law spectrum and continuity of the magnetic field across the shock, involving the respective matching conditions at the shock.

Below the upstream (downstream) quantities are labeled with the index ‘1’ (‘2’).

## 2. Simulations

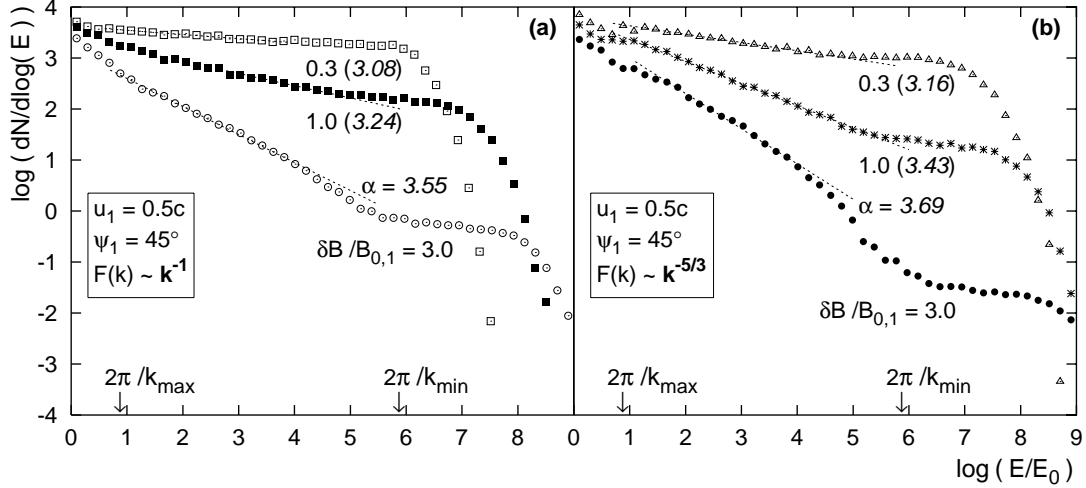
In the simulations trajectories of ultrarelativistic test particles are derived by integrating their equations of motion in the perturbed magnetic field [for details see: Niemiec, Ostrowski, in preparation]. We consider a relativistic planar shock wave propagating in rarefied electron-proton plasma. Upstream of the shock the field consists of the uniform component,  $B_{0,1}$ , inclined at some angle  $\psi_1$  to the shock normal and finite-amplitude perturbations imposed upon it. The perturbations are modeled as a superposition of 294 sinusoidal static waves of finite amplitudes [cf. 10] which have either a flat ( $F(k) \sim k^{-1}$ ) or a Kolmogorov ( $F(k) \sim k^{-5/3}$ ) wave power spectrum in the (wide) wave vector range ( $k_{min}, k_{max}$ ) and  $k_{max}/k_{min} = 10^5$ . The shock moves with the velocity  $u_1$  with respect to the upstream plasma. The downstream flow velocity  $u_2$  and the magnetic field structure are obtained from the hydrodynamic shock jump conditions, so that the field is continuous across the shock. Derivation of the shock compression ratio defined in the shock rest frame as  $R = u_1/u_2$ , is based on the approximate formulae derived in Ref. [5]. We consider the acceleration process in the particle energy range where radiative (or other) losses can be neglected.

## 3. Results

In Fig. 1 we present particle spectra for the oblique subluminal ( $u_{B,1} \equiv u_1/\cos\psi_1 < c$ ) shock wave with  $u_1 = 0.5c$  and  $\psi_1 = 45^\circ$ . The shock velocity along the mean magnetic field is then  $u_{B,1} = 0.71c$  and the shock compression ratio is  $R = 5.11$ . The particle spectra are measured at the shock for three different magnetic field perturbation amplitudes and the flat (Fig. 1a) and the Kolmogorov (Fig. 1b) wave power spectra. The following features are visible in the spectra:

- the particle spectra diverge from a power-law in the full energy range;
- before the spectrum cut-off a harder spectral component can appear;
- the exact shape of the spectrum depends on both the amplitude of the magnetic field perturbations and the wave power spectrum.

One may note that a power-law part of the particle spectrum steepens with increasing amplitude of the field perturbations, more for the Kolmogorov perturbations. The obtained spectral indices are consistent with previous numerical

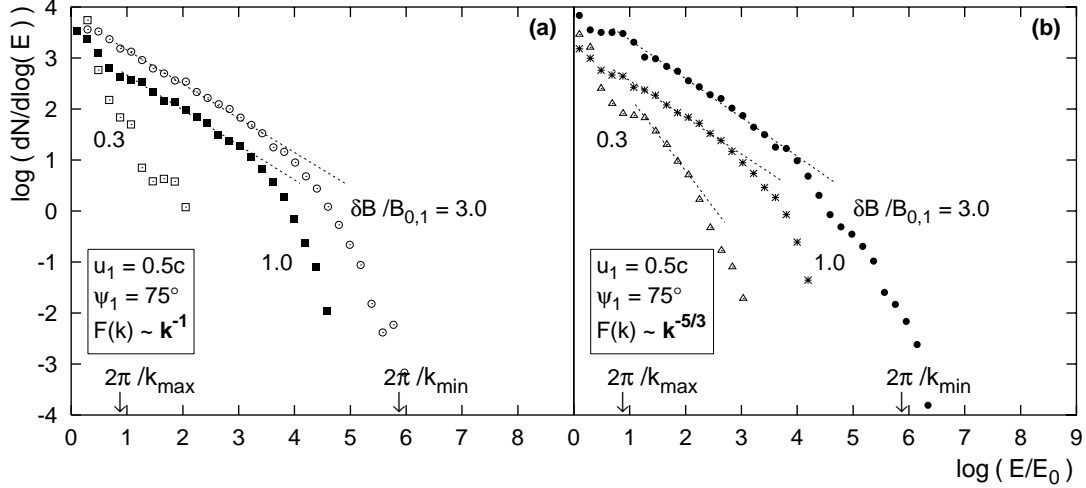


**Fig. 1.** Accelerated particle spectra at the subluminal shock wave ( $u_1 = 0.5c$ ,  $\psi_1 = 45^\circ$  and  $u_{B,1} = 0.71c$ ) for (a) the flat ( $F(k) \propto k^{-1}$ ) and (b) the Kolmogorov ( $F(k) \propto k^{-5/3}$ ) wave spectrum of magnetic field perturbations. The upstream perturbation amplitude  $\delta B/B_{0,1}$  is given near the respective results. Linear fits to the power-law parts of the spectra are presented and values of the phase space distribution function spectral indices  $\alpha$  are given. Particles of energies in the range indicated by arrows can effectively interact with the magnetic field inhomogeneities ( $k_{min} < k_{res} < k_{max}$ ).

calculations [9, 10] and the analytic results obtained in the limit of small perturbations [6].

The non power-law character of the obtained particle spectra results from the limited dynamic range of magnetic field perturbations. In the energy range where the approximately power-law spectrum forms particles can be effectively scattered by the magnetic field inhomogeneities. The character of the spectrum changes at highest particle energies where  $k_{res} \leq k_{min}$  and particles are only weakly scattered. Then the anisotropically distributed upstream particles can effectively reflect from the region of compressed magnetic field downstream of the shock leading to the spectrum flattening [cf. 9]. The cut-off in the spectrum is formed mainly due to very weakly scattered particles escaping from the shock to the introduced upstream free escape boundary.

The spectra obtained for superluminal shocks with  $u_{B,1} \approx 2$  are presented in Fig. 3. For the low amplitude turbulence ( $\delta B/B = 0.3$ ) we approximately reproduce results of Ref. [4], with a ‘super-adiabatic’ compression of injected particles, but hardly any power-law spectral tail. At larger turbulence amplitudes power-law sections in the spectra are produced again, but the steepening and the cut-off occur at lower energies in comparison to the subluminal shocks (Fig. 1).



**Fig. 2.** Accelerated particle spectra at the superluminal shock ( $u_{B,1} = 1.93c$ ) for (a) the flat and (b) the Kolmogorov spectra of magnetic field perturbations.

#### 4. Summary

The present work is intended to study the first-order Fermi acceleration process acting at relativistic shocks. In comparison to the previous work we include a few ‘realistic’ features of the considered turbulence. We show that the spectrum can deviate from the usually considered power-law form and we demonstrate variation of the particle spectral index for its power-law section with the spectrum and amplitude of turbulence and the mean field inclination.

The work was supported by the Polish State Committee for Scientific Research in 2002-2004 as a research project 2 P03D 008 23 (JN) and in 2002-2005 as a solicited research project PBZ-KBN-054/P03/2001 (M0).

#### References

1. Ballard K.R., Heavens A. 1992, MNRAS 259, 89
2. Bednarz J., Ostrowski M. 1996, MNRAS 283, 447
3. Bednarz J., Ostrowski M. 1998, Phys. Rev.Lett. 80, 3911
4. Begelman M.C., Kirk J.G. 1990, ApJ 353, 66
5. Heavens A., Drury L’O.C. 1988, MNRAS 235, 997
6. Kirk J.G., Heavens A. 1989, MNRAS 239, 995
7. Kirk J.G., Schneider P. 1987, ApJ 315, 425
8. Kirk J.G., Schneider P. 1987, ApJ 322, 256
9. Ostrowski M. 1991, MNRAS 249, 551
10. Ostrowski M. 1993, MNRAS 264, 248