# Second-order Fermi Acceleration in the Interstellar Medium and its Effects on Cosmic-ray Electrons

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## Abstract

We investigated the diffusive reacceleration of cosmic rays in the interstellar space presented by Seo&Ptuskin(1994) and Heinbach&Simon(1995) in which different formulations are used. It becomes clear that the difference between their reacceleration coefficients is a factor of 11/6 (= $(2-\alpha/2)$ with a Kolmogorov spectrum) for 1GeV cosmic rays. There is, however, little difference in the total energy gain of cosmic rays passing through the Galaxy. Using the same parameters as in the nuclei propagation, we calculated the reacceleration model for electrons.

## 1. Introduction

Second-order Fermi acceleration of cosmic rays in the interstellar medium is called diffusive reacceleration and generally described as diffusion in the momentum space[7](Ptuskin). And there is another kind of formulation given by Simon et al.[1](Simon) that defines the reacceleration parameter directly in the Fokker-Planck equation. We give the analytical solutions for two different formulations using Green's function and calculate the mean energy gain rate per g/cm$^2$ in order to apply it to the cosmic-ray electron propagation.

## 2. Solutions for Second-order Fermi Acceleration

The second-order Fermi acceleration process is represented by the diffusion in momentum space and the distribution $f(x,p)$ after $x$g/cm$^2$ satisfies

$$\frac{\partial f(x,p)}{\partial x} - \frac{1}{2}\frac{1}{\beta p^2}\frac{\partial}{\partial p}(p^4 \frac{1}{\beta}\sigma X_e \frac{\partial f(x,p)}{\partial p}) = 0 \ ,$$

where the escape length $X_e = X_0 R^{-\alpha} = \mu\beta cH/(2D)$ (rigidity $R$). The parameter $\sigma(\mathrm{g/cm^2})^{-2} = C(\alpha)h_a/H(v_a/\mu c)^2$ with the halo size $H$, reacceleration space $h_a$, Alfven speed $v_a$, the surface density of galactic disk $\mu$ and $C(\alpha) = 32/3\alpha(4-\alpha^2)(4-\alpha) \simeq 2.2$ [7][2]. The reacceleration parameter $\eta_p$ is defined by

$$\sigma X_e \equiv \eta_p R^{-\alpha} = C(\alpha)\frac{h_a}{2\mu}\frac{v}{D}(\frac{v_a}{c})^2 \ ,$$

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with the spatial diffusion coefficient $D = D_0 \beta R^\alpha$ ($\beta = v/c$). When $\beta \sim 1$, we get the Green's function with $\nu = -1 + 3/\alpha$, $\eta' = \eta_p z^\alpha/2$ (charge $z$) and $P = pc$,

$$G(P,P',u) = \frac{2}{\eta'\alpha}(PP')^{-\alpha\nu/2} I_\nu(2\sqrt{\frac{u}{\eta'\alpha^2}P_<^\alpha}) K_\nu(2\sqrt{\frac{u}{\eta'\alpha^2}P_>^\alpha}) \tag{1}$$

where $u$ is the Laplace transform variable, $I_\nu$ and $K_\nu$ are the modified Bessel function of the first kind and second kind respectively (e.g. R.Schlickeiser 2000). $U_<$ means the smaller of energy $U$ and $U'$, and $U_>$ is the larger. The solution $N(E,x) = p^2 f(p,x)$ with the initial function $N(P,0)$ is

$$N_\nu(E,x) = \frac{P^{\alpha\nu/2+\alpha-1}}{x\eta'\alpha}\int_0^\infty N(P',0)P'^{-\alpha\nu/2}\exp(-\frac{P^\alpha + P'^\alpha}{x\eta'\alpha^2}) I_\nu(\frac{2(PP')^{\alpha/2}}{x\eta'\alpha^2}) dE' \ . \tag{2}$$

Starting with mono energy $N(P,0) = \delta(P - P_0)$, eq. (2) reduces to

$$N_\nu(E,x) \ [\mathrm{GeV}^{-1}] = \frac{P^{\alpha\nu/2+\alpha-1}P_0^{-\alpha\nu/2}}{x\eta'\alpha}\exp(-\frac{P^\alpha + P_0^\alpha}{x\eta'\alpha^2}) I_\nu(\frac{2}{x\eta'\alpha^2}(PP_0)^{\alpha/2}) \ . \tag{3}$$

In the case of Simon[1], the coefficient of the Fokker-Planck equation is directly associated with the reacceleration parameter $\eta(\mathrm{g/cm^2})^{-1}$.

$$\frac{\partial N(E,x)}{\partial x} + \frac{\partial}{\partial E}[\eta \cdot E \ \ R^{-\alpha} N(E,x)] - \frac{1}{2}\frac{\partial^2}{\partial E^2}[\eta \cdot E^2 \ \beta^2 \ R^{-\alpha} N(E,x)] = 0 \ .$$

The Green's function and the solution of this equation are given by eq. (1) and eq. (2) with $\nu = 1/\alpha$ and $P \to E$(total energy). The solution of mono energy $N(E,0) = \delta(E - E_0)$ is given by $N_{1/\alpha}(E,x)$ in eq. (3) and exactly generates the curves shown in Simon et al.(Fig.3 in [8]). Next we calculate the energy gain rate using eq. (3). Cosmic rays with the initial energy $E_0$ have the mean energy $E_{av}(E_0,x) = E_0 y^{-1/\alpha}\ (\Gamma(A)/\Gamma(B))(F(A,B;y)/e^y)$ after $x\mathrm{g/cm^2}$, where $\Gamma$ is the gamma function and $F(A,B;y)$ is the confluent hypergeometric function with

$$(A,B,y) = (\ \frac{4}{\alpha},\ \frac{3}{\alpha}, \frac{2P_0^\alpha}{x\eta_p\alpha^2})(Ptuskin),\ (A,B,y) = (\ 1{+}\frac{2}{\alpha},\ 1{+}\frac{1}{\alpha}, \frac{2E_0^\alpha}{x\eta\alpha^2})(Simon) \ .$$

Actually in the case of $\eta = \eta_p = 0.06$, 1GeV/n Carbon increases the energy of 63% (Ptuskin) and 32% (Simon) after $x$ =5g/cm$^2$. The energy gain rate is

$$g_E \equiv \frac{1}{E_0}\frac{dE_{av}(E_0,x)}{dx} = \frac{1}{\alpha\ x} y^{-1/\alpha}\frac{\Gamma(A)}{\Gamma(B)}\frac{F(A-1,B;y)}{e^y} \ .$$

$F(A{-}1,B;y)$ can be asymptotically expanded since $y$ is always a large value. The first term becomes $g_E \sim (2-\alpha/2)\eta_p P_0^{-\alpha}$ in the case of Ptuskin, that is sufficiently accurate. In the case of Simon, of course, the first term is $g_E \sim \eta E_0^{-\alpha}$. There is a

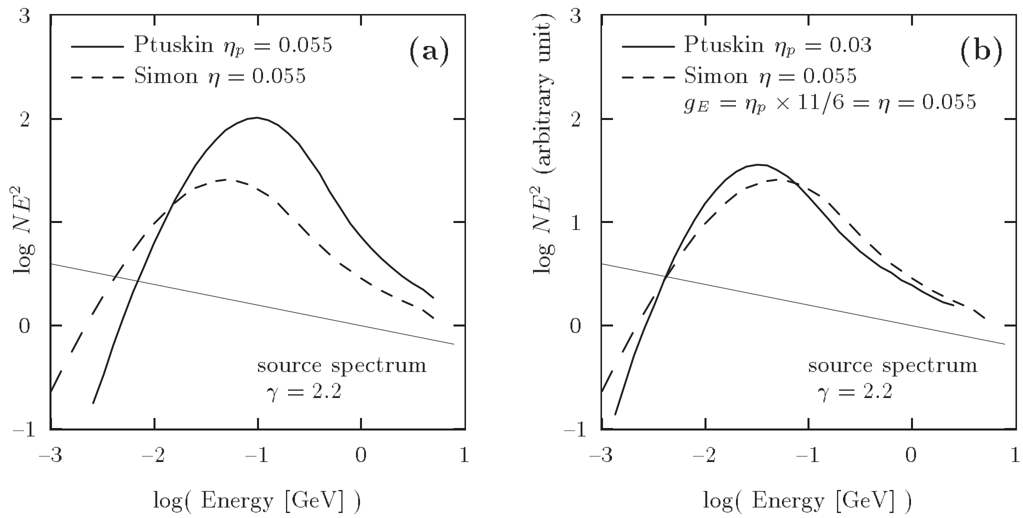


**Fig. 1.** Eq.(2) with the initial single power-law spectrum, (a) the same reacceleration parameters $\eta = \eta_p$, (b) the same energy gain rates $g_E(1\text{GeV}) = \eta = \eta_p(2 - \alpha/2)$.

factor of $(2-\alpha/2) = 11/6$ difference between the two parameters if $\alpha = 1/3$. The results of the power-law spectrum are shown in Fig.1, where $N_\nu(E, x)$ is given by eq. (2) with the source spectrum $N(E, 0) = E^{-\gamma}\epsilon(E - E_{min})$ with the minimum energy $E_{min} = 10^{-3}$GeV, ($\epsilon(x)$ : the Heaviside function). Fig.1 shows that the spectrum for Ptuskin raises more than that for Simon if $\eta_p = \eta$ (Fig1.a), while the two spectra are almost identical for the same energy gain $g_E$ (Fig1.b).

## 3. Propagation Calculations

First we investigated the parameters that are estimated from the observed B/C and sub-Fe/Fe ratios and primary nuclei spectra in each case. The total energy gain, that is, $g_E(\text{g/cm}^2)^{-1}$ multiplied by the escape length $X_0(\text{g/cm}^2)$ at 1GeV/n, is $g_E \cdot X_0 = (2-\alpha/2) \cdot \eta_p \cdot X_0 = 1.83 \cdot 0.022 \cdot 9.4 = 0.38$ in Ptuskin(2001)[6] and $g_E \cdot X_0 = \eta \cdot X_0 = 0.064 \cdot 6.4 = 0.41$ in Simon[4]. Each estimate gives the almost same result of 40% energy gain at the 1GeV/n cosmic ray in the interstellar space. Next we investigate the effects of reacceleration on cosmic-ray electrons using the parameters estimated from nuclei data (the parameters in Ptuskin is employed in Fig.2). The energy loss processes of cosmic-ray electrons shown in Fig.2(a) are Ionization $1.9 \times 10^{-16} n_H(1 + 0.146 \ln(E/m)$ [GeV/sec] with $n_H = 0.3$, Bremsstrahlung $1.02 \times 10^{-15} n_H E$ [GeV/sec], Escape $E/(2 \times 10^7 E^{-1/3})$ [GeV/yr] and Synchrotron + Inverse Compton $2.0 \times 10^{-16} E^2$ [GeV/sec]. The reacceleration rate shown in Fig.2(a) is nearly proportional to $E^{1-\alpha}$ and dominates the range of $10^{-(2\sim1)}$ GeV. Fig.2(b) shows the curves of leaky box model(LBM) including all processes in Fig.2(a) with the electron measurements. As observed intensities below 10GeV are influenced by the solar modulation, we fit the model to the

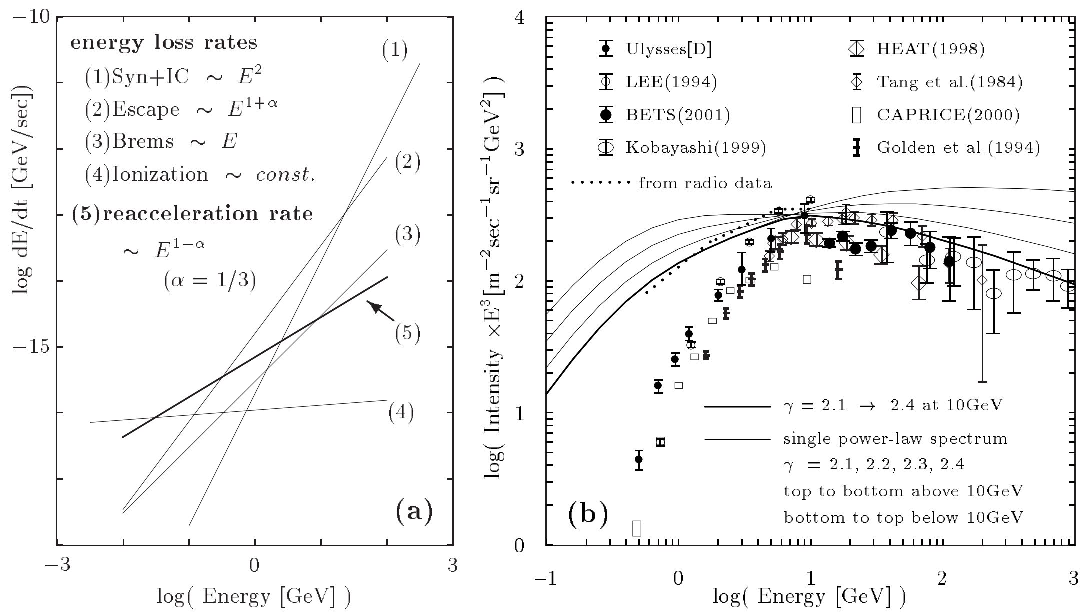


**Fig. 2.** (a) Energy change processes for cosmic ray electrons in the interstellar space (b) LBM with reacceleration having single power-law source spectra(thin lines) compared with electron measurements. The bending thick line is fitted to the radio data.

spectrum estimated from the radio data[3]. The result shows that the source spectrum has a break around 10GeV with the change of spectral index from –2.1 to –2.4, that has been shown in Moskalenko et al.(1998)[5].

## 4. Conclusions

We have shown the difference in reacceleration parameter between the two formulations [1][7]. Those formulations, however, give almost the same energy gain in the nuclei propagation. If the same parameters as estimated from nuclei data are applied to the electron propagation, the effects of reacceleration appears below 1GeV and needs the break in the source spectrum as indicated previously.

## References

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