The growth of Parker instability with the effect of cosmic-ray diffusion

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Abstract

Two-dimensional magnetohydrodynamic (MHD) simulation of a galactic Parker instability including the effect of an anisotropic Cosmic-Ray (CR) diffusion is performed. In this simulation, we take a two-temperature layered disk model which is composed of gas, magnetic field, and cosmic rays in uniform gravity. A mechanical perturbation case and an explosion case are investigated. In the case of mechanical perturbation case, the growth rate becomes smaller as the diffusion coefficient of CR decreases. In the explosion case, when the diffusion coefficient is small, the growth rate is large in early stage, but it slows down after the top of the loop passes the disk surface.

1. Introduction

The CR in interstellar medium (ISM) play an essential roles in the dynamics of ISM because it is recognized that the energy density of cosmic rays is the same order as that of magnetic field and turbulent gas motions [5]. There are many works about nonlinear evolution of Parker instability and Parker-Jeans instability without the effect of CR [1, 4, 6]. On the other hand, in spite of the suggestions of many astrophysical applications, there are few papers on the nonlinear evolution of Parker instability with the effect of CR. Hanasz et al. [3] carried out calculations of Parker instability induced by CR injection from a supernova under the thin-flux-tube approximation. But, still, there are many unknown about the effect of CR for Parker instability.

In this Paper, we present the results of Parker instability with the effect of CR diffusion by two-dimensional MHD simulations starting from an equilibrium two temperature layered MHD disk.
2. Numerical Simulations

The basic equations we use are ideal two-dimensional MHD equations and cosmic ray energy equation in the Cartesian coordinate system \((x, z)\), where we adopted approximation \(\hat{x} = \hat{\varphi}\) and \(\hat{z} = \hat{z}\) on cylindrical coordinate system \((r, \varphi, z)\) of galactic disk. The initial condition is an equilibrium model under the two temperature layered disk model, \(T(z) = T_0 + (T_{\text{halo}} - T_0)^{1/2}(\tanh((z - z_{\text{halo}})/w_{\text{tr}}) + 1)\), where the disk temperature is \(T_0 = 10^4\) K, the halo temperature is \(T_{\text{halo}} = 25 \times 10^4\) K, the height of the disk-halo interface is \(z_{\text{halo}} = 900\) pc, and the width of the transition layer is \(w_{\text{tr}} = 30\) pc. The magnetic fields are horizontal initially. The total gas pressure scale height at \(z = 0\) (equatorial plane of galactic disk) is \(H = (1 + \alpha + 1/\beta)C_{s0}^2/(\gamma_g g_z)\), where \(\alpha, \beta, C_{s0}, \gamma_g\) and \(g_z\) are the ratio of CR pressure to gas pressure, the plasma \(\beta\), the sound velocity at \(z = 0\), the adiabatic indices for gas and the gravitational acceleration. Here, we set \(\alpha = 1, \beta = 1\) initially and \(\gamma_g = 1.05, g_z = \text{constant in every time.}\) The system is initially homogeneous in the \(x\)-direction. For normalization, we take the length \(H_0 = C_{s0}^2/(\gamma_g g_z)\) as unit length and set \(\rho_0 = C_{s0} = 1\). We assume that the cosmic ray diffusion takes place only along magnetic field lines because the ratio of parallel to perpendicular diffusion is very small, 0.02-0.04 [2]. In this assumption, the cosmic ray energy equation becomes as follows:

\[
\frac{\partial}{\partial t} \left( \frac{P_c}{\gamma_c - 1} \right) + \nabla \cdot \left( \frac{\gamma_c}{\gamma_c - 1} P_c \right) V - V \cdot \nabla P_c - \nabla \cdot \left[ \kappa || \nabla || \left( \frac{P_c}{\gamma_c - 1} \right) \right] = 0, \tag{1}
\]

where \(\gamma_c = 4/3\) is the adiabatic indices for cosmic ray and the subscript || means taking the value which is the component parallel to the magnetic field. For solving the diffusion part in this equation, we used an implicit method. We calculate only the region over the equatorial plane of galactic disk. We take two types of perturbation, the mechanical perturbation and the explosion perturbation. In the case of mechanical perturbation, we add small velocity perturbations. In the case of explosion, an energy of cosmic ray (\(\sim 10^{50}\) ergs) is put into a region with volume \(V_{\text{exp}} = \pi r_{\text{exp}}^2 y_{\text{exp}}\), where \(r_{\text{exp}} = 25\) pc, \(y_{\text{exp}} = 50\) pc.

Figure 1a shows the initial conditions of mechanical perturbation case. The gray scale contour denotes CR pressure distribution, white lines are magnetic field lines and the arrow at the upper right corner shows a reference velocity vector (= \(10 \times C_{s0}\)). We calculated three models for different normalized CR diffusion coefficient values, \(\kappa = 200 \ (\sim 3 \times 10^{28} \text{ cm}^2 \text{ s}^{-1} \text{ in real scale}), \kappa = 20\) and \(\kappa = 2\). Figure 1b, 1c and 1d show the growth of the system in the case of \(\kappa = 200\) at \(t = 27t_0\), \(\kappa = 20\) at \(t = 33t_0\) and \(\kappa = 2\) at \(t = 67t_0\), where \(t_0\) is the sound crossing time \(t_0 = H_0/C_{s0} \ (\sim 5\) Myr in real scale). The disk matter falls down along magnetic field and the evacuated region goes up to the halo region by magnetic buoyancy. The differences of morphology, the shape of magnetic loop,
the CR pressure distribution, the velocity field are very small between the cases of $\kappa = 200$ and $\kappa = 20$. On the other hand, the system drastically changes in the case of $\kappa = 2$. The shape of magnetic loop looks like being pulled in right and left sides and the CR pressure mainly distributes in the bottom region of magnetic loop. Conversely, the CR pressure mainly distributes along magnetic field lines in other models. The growth rate becomes smaller as the diffusion coefficient of CR decreases. In figure 2, the upper panels show the CR pressure distribution

![Figure 1](image1)

**Fig. 1.** The initial CR pressure distribution (a) and the CR pressure distributions for different values of CR diffusion coefficient (b) $\kappa = 200$ at $t = 27t_0$, (c) $\kappa = 20$ at $t = 33t_0$, (d) $\kappa = 2$ at $t = 67t_0$.

![Figure 2](image2)

**Fig. 2.** The CR pressure value (bottom panel) along a magnetic field line (a white curve in upper panel) (a) $\kappa = 200$ at $t = 28t_0$, (b) $\kappa = 20$ at $t = 34t_0$, (c) $\kappa = 2$ at $t = 50t_0$.

(gray scale), velocity vectors and a magnetic field line (white curve) in each case, (a) $\kappa = 200$ at $t = 28t_0$, (b) $\kappa = 20$ at $t = 34t_0$, (c) $\kappa = 2$ at $t = 50t_0$ when the difference of CR pressure value between at the loop top and footpoint becomes maximum. The bottom panels show the CR pressure value along a magnetic field line depicted in upper panels. The difference of CR pressure value decreases as the $\kappa$ increases because the propagation speed of CR energy is larger and the distribution approaches to uniform quickly along a magnetic field as $\kappa$ becomes large. In figure 2c, this value is very large even in early stage for the growth of
instability. Figure 3 shows the result of explosion case, (a) $\kappa = 20$, (b) $\kappa = 2$ at $t = 24t_0$. In late stage, $t = 24t_0$, the growth speed of instability in $\kappa = 20$ is faster than that in $\kappa = 2$. On the other hand, the growth is faster in $\kappa = 2$ than that in $\kappa = 20$ in early stage when the region in which the explosion energy was put locates inside of the disk.

3. Discussion

The mechanical perturbation case and the explosion case were investigated. The growth speed of instability is larger as the CR diffusion coefficient $\kappa$ increases in both case. The difference of CR pressure value between at the loop top and the footpoint becomes large as the $\kappa$ decreases. When the difference of CR pressure is large, the CR pressure gradient force becomes large and it prevent from the gas to slides down along magnetic loop and the growth rate of instability becomes small. In the explosion case, Hanasz et al. [3] showed that smaller values of $\kappa$ lead to shorter timescales of the instability because the smaller the diffusion coefficient, the longer cosmic ray remain in the close neighborhood of the injection region and dominate the local dynamics. We got same result in early stage. But, finally, the result becomes same as that of mechanical perturbation case.

We thank K. Shibata for useful discussion. Numerical computations were carried out on VPP5000 at NAOJ. TK and CMK are supported in part by the National Science Council, Taiwan, under the grants NSC-91-2112-M-008-006, NSC-90-2112-M-008-020, NSC-91-2112-M-008-050.

4. References

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