Propagation of Radioactive Secondaries in Cosmic Rays

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Abstract

Based on the analytical solution of the cosmic-ray propagation recently we have obtained, we apply it for the radioactive secondaries, such as ^{10}Be. We assume our Galaxy is boundaryless in both longitudinal and the latitudinal directions, and the diffusion coefficient depends on the rigidity in the form of $\propto vR^{\alpha}$, and also on the spatial coordinate $\boldsymbol{r}(r,z)$ in the form of the exponential type.

We compared the present numerical results with those obtained by recent experiments, and found the Galactic parameters thus estimated, such as the diffusion coeffeicient and the gas density are consistent with those expected from B/C and/or sub-Fe/iron ratio, and the diffused γ-rays.

1. Introduction

The radioactive component with the life time comparable with the cosmic-ray residence time in the Galaxy brings us a critical information for the cosmic-ray propagation as well as the structure of the Galaxy. Typical component is ^{10}Be with the life time of 2.18×10^6 y. The unstable nucleus component can not stay far from the Galactic plane due to the limited life time, and thus the propagation history is quite different from those of the stable ones. So, we can get additional parameters, for instance the gas density, which is difficult to estimate from the stable component alone.

In Papers I, II[1, 2], we have derived the solution of the cosmic-ray propagation for the primary and the secondary components, both of which are of course stable. As was mentioned in these papers, in order to estimate many parameters appearing in this model, we need more additional components such as diffused γ-ray, radioactive component, $\bar{\mathrm{p}}$ and so on.

In the present paper, we show analytical solution for the unstable component, and compare the numerical results with ^{10}Be/^{9}Be data. Because of limited space, we show only the solution without the low energy effect such as ionization loss and the reacceleration, while we touch briefly the solution with the low energy effect.

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2. Solution of the Diffusion Equation for Unstable Nucleus

If we don't take into account the energy change during the propagation, it is straightforward to write down the diffusion equation, i.e., Eq. (4) in Paper I is replaced by

$$\left[\nabla\cdot D(\boldsymbol{r})\nabla - n(\boldsymbol{r})v\sigma_\tau - \frac{1}{\tau_0\mathit{\Gamma}}\right]\mathit{\Phi}_\tau(\boldsymbol{r};\boldsymbol{r}_0) = -\frac{\delta(\boldsymbol{r}-\boldsymbol{r}_0)}{2\pi r_0}, \tag{1}$$

where σ_τ and τ_0 are the inelastic cross section and the life time of the unstable nucleus, respectively, and $\mathit{\Gamma}$ its Lorentz factor. All variables used in the present paper are the same as those defined in Papers I, II. In the present paper, we often meet a dimensionless parameter η_0

$$\eta_0 = 2z_D/\sqrt{\tau_0 D_0}. \tag{2}$$

It is a little bit cumbersome to solve analytically Eq. (1), but remarking the fact $\nu \approx 0.1-0.3$ estimated from the data of 2-ry/1-ry ratio, we can solve Eq. (1) in the form of the expansion with respect to ν. Once we find a Green function $\mathit{\Phi}_\tau$, we can obtain straightforwardly the density of the unstable secondary component τ, similarly as in the case of the stable one (see Eq. (1) in Paper II),

$$N_{p\to\tau}(\boldsymbol{r}) = \iint d\boldsymbol{r}_0[N_p(\boldsymbol{r}_0)n(\boldsymbol{r}_0)v\sigma_{p\to\tau}]\,\mathit{\Phi}_\tau(\boldsymbol{r};\boldsymbol{r}_0), \tag{3}$$

where $\sigma_{p\to\tau}$ is the production cross section of the unstable nucleus due to the interaction of the primary nucleus p with the interstellar gas. Here we omit the rigidity term R_0 $(=R)$ for the simplicity.

Let us write down the solution including the rigidity term R at the Galactic plane explicitly,

$$\frac{N_{p\to\tau}(r;R)}{N_p(r;R)} = 4\nu^2R^{-\alpha}\frac{\sigma_{p\to\tau}}{\sigma_r}\frac{\boldsymbol{\mathcal{I}}_{\nu,\eta}(U_{r,R};\tilde{U}_{r,R})}{\tilde{U}_{r,R}\boldsymbol{I}_{\eta-1}(\tilde{U}_{r,R})}, \tag{4}$$

$$\text{with}\quad \frac{\eta}{\nu} = \sqrt{1+\frac{c\eta_0^2}{v\mathit{\Gamma}}},\quad \tilde{U}_{r,R} = U_{r,R}\sqrt{\frac{\sigma_\tau}{\sigma_p}}. \tag{5}$$

Eq. (4) is completely the same form as the solution for the stable one given by Eq. (2) in Paper II, except the functions, $\boldsymbol{\mathcal{I}}_{\nu,\eta}$ and $\boldsymbol{I}_{\eta-1}$. Here we introduced

$$\boldsymbol{I}_\eta(U) = I_{\eta,0}(U) + \eta_D^2 I_{\eta,1}(U) + \eta_D^4 I_{\eta,2}(U) + \dots, \tag{6}$$

$$\boldsymbol{\mathcal{I}}_{\nu,\eta}(a;\tilde{a}) \simeq \int_0^1 t[1+\nu(1-t)+\dots]\boldsymbol{I}_\eta(\tilde{a}t)dt, \tag{7}$$

$$\text{with}\quad I_{\eta,0}(U) \equiv I_\eta(U), \tag{8}$$

where

$$\frac{\eta_D}{\nu} = \sqrt{\frac{c\eta_0^2}{v\mathit{\Gamma}}}\left(\frac{1}{2\nu}\sqrt{\frac{\sigma_0}{\sigma_\tau}}\right)^\nu,\quad \text{with}\quad \sigma_0 = \frac{D_0}{n_0 c z_D^2}, \tag{9}$$

$$I_{\eta,1}(U) = \int_0^U (X^{2\nu} - 1)[I_\eta(U)K_\eta(X) - I_\eta(X)K_\eta(U)]I_{\eta,0}(X)dX. \qquad (10)$$

$I_\eta(U), K_\eta(U)$ are the modified Bessel functions with the index of η, and the higher order term is similarly obtained by replacing $(I_{\eta,1}, I_{\eta,0})$ in Eq. (10) into $(I_{\eta,2}, I_{\eta,1})$ respectively. Practically, however, we find only the second contribution is enough in the case of $\nu \lesssim 0.3$.

Now, we have already obtained the solution of the stable secondary component in Paper II, and thus the relative intensity of the unstable component to the stable one is immediately written down as

$$\frac{N_{p\to\tau}(r;R)}{N_{p\to s}(r;R)} = \frac{\sigma_{p\to\tau}}{\sigma_{p\to s}}\sqrt{\frac{\sigma_s}{\sigma_\tau}}\frac{I_{\nu-1}(\hat{U}_{r,R})}{\boldsymbol{I}_{\eta-1}(\tilde{U}_{r,R})}\frac{\boldsymbol{\mathcal{I}}_{\nu,\eta}(U_{r,R};\tilde{U}_{r,R})}{\mathcal{I}_{\nu,\nu}(U_{r,R};\hat{U}_{r,R})}. \qquad (11)$$

In the above solution, we omit the mass difference between the stable and the unstable components for the sake of simplicity, but the effect is negligible in comparison with the statistical error in the experimental data, for instance ^{10}Be/^{9}Be nowadays available.

For extreme case of the energy (rigidity), we find a reasonable result

$$\frac{N_{p\to\tau}(r;R)}{N_{p\to s}(r;R)} \approx \frac{\sigma_{p\to\tau}}{\sigma_{p\to s}} \times \begin{cases} 1, & \text{for } R \to \infty, \qquad (12a) \\ \dfrac{1}{\eta_0}\sqrt{R^\alpha \dfrac{v}{c}}, & \text{for } R \to 0. \qquad (12b) \end{cases}$$

For $R \to 0$, however, we can not apply the above result since we neglect the low energy effect, and it must be replaced by $(v/c)/\eta_0^2$. Practically, however, taking account of the solar modulation effect, the ratio becomes constant in the low energy, say $\lesssim$100 MeV, where the constant (saturated) value corresponds approximately to the ratio at the modulation energy $Ze\Phi/A$.

In the present paper, we have no space to give the solution including the low energy effect such as the ionization loss and the reacceleration, but it is possible to obtain analytically the solution with use of the technique developed by Ptsukin et al.[3], where they showed that the weighted slab approximation can give exact solution if the diffusion coefficient is a separable function of position $\boldsymbol{r}$ and the rigidity R, just corresponding to our assumption on the diffusion coefficient

$$D(\boldsymbol{r};R) = D_0 v R^\alpha \exp[+(r/r_D + |z|/z_D)] = vR^\alpha D(\boldsymbol{r}). \qquad (13)$$

3. Numerical results and discussion

In this paper, we compare the numerical results of Eq. (11) for ^{10}Be/^{9}Be with the experimental data, while those including the low energy effect will be reported in the conference if in time. Before going to the numerical calculations,

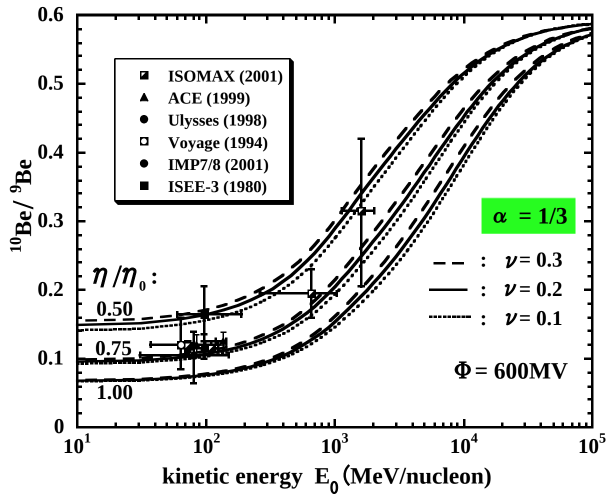

Fig. 1. Abundance ratio of ^{10}Be to ^{9}Be.

we first give σ_0 and η_0, appearing often in the present work, for typical values of the propagation parameters: D_0=10^{28}cm^2, n_0=1cm^{-3}, z_D=1Kpc, and τ_0=$2.18 \cdot 10^6$y in the case of ^{10}Be,

$$\sigma_0 = 34.94\text{mb}, \quad \text{and} \quad \eta_0 = 7.46. \tag{14}$$

In the following discussion, we measure σ_0 and η_0 in unit of the above values.

In Fig. 1 we demonstrate the result, where we showed several cases of the parameters, ν=0.1, 0.2 and 0.3, and η/η_0=0.50, 0.75, and 1.00, while σ_r/σ_0 is fixed to 1.0 since the ratio of our interest depends weakly on σ_r. We found the numerical curves reproduce nicely if we choose appropriate set of parameters. Full consideration about the choice of the parameters will be reported in the conference in connection with other data, such as primary spectrum[1], 2-ry to 1-ry ratio[2], and diffused γ-ray[4].

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