Cosmic-ray Propagation and the Energy Spectra Observed on Earth

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Abstract

We assume that our Galaxy is boundaryless, and both the diffusion coefficient and the gas density depend on the spatial coordinate $\boldsymbol{r}(r, z)$ in the form of the exponential type, and we take the rigidity dependence of the diffusion coefficient into account, having a form $D_0(R) \propto vR^{\alpha}$. We assume further that the source spectrum has a power-like form in rigidity, $\propto R^{-\gamma}$, and the spatial distribution is also of the exponential type.

We present an analytical solution, and compare it with recent data obtained by BESS, RUNJOB and others.

1. Introduction

It is quite important to build the unified picture for all observables in cosmic ray components, and many people have studied the cosmic-ray propagation from various point of view[1]. In the present paper, we show an analytical approach to these problems in three dimensional way, taking the realistic structure of the Galaxy into account as presented in the next section, and apply it to the primary components, such as proton $\sim$ iron, recently obtained by BESS[2], RUNJOB[3] and others[4]. Secondary components (LiBeB and sub-iron), radioactive one (^{10}Be, ^{26}Al, ...) and the diffused γ-rays are presented separately in this proceeding[5, 6, 7].

2. Basic Assumption

We assume that the diffusion coefficient, the gas density, and the source density of CR, depend on the coordinate $\boldsymbol{r}(r, z)$ with cylindrical symmetry

$$D(\boldsymbol{r}) = D_0 \exp[+(r/r_D + |z|/z_D)], \qquad (1a)$$

$$n(\boldsymbol{r}) = n_0 \exp[-(r/r_n + |z|/z_n)], \qquad (1b)$$

$$Q(\boldsymbol{r}) = Q_0 \exp[-(r/r_Q + |z|/z_Q)], \qquad (1c)$$

where D_0, n_0 and Q_0 correspond to diffusion coefficient, gas density and the source density of CR at the Galactic center $(0, 0)$, respectively.

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We further assume that D_0 and Q_0 have following rigidity dependence,

$$D_0(R) \Rightarrow D_0 v R^{\alpha}, \qquad Q_0(R) \Rightarrow Q_0 R^{-\gamma}. \tag{2}$$

In the present paper, however, we focus on the diffusion problem in the high energy region only ($v \approx c$), $\gtrsim 5\,\mathrm{GeV}$, where the energy change is negligible, namely the rigidity R at the observation point $\boldsymbol{r}$ equals that at the source. So the diffusion coefficient $D(R;\boldsymbol{r})$ is often written simply as $D(\boldsymbol{r})$, omitting the rigidity R. The diffusion process with the energy change will be reported elsewhere near future.

Another important assumtion relates to the configuration of our Galaxy. While past works have set the boudary in both longitudinal and the latitudinal spreads with, for instance, $15 \sim 20\,\mathrm{kpc}$ and $2 \sim 3\,\mathrm{kpc}$ respectively, we don't set them here, but introduce the scale-height parameters as appearing in equations (1a)-(1c).

In the prsent work, the ratio of z_D to z_n and the average of these two scale heights play essential role, and so we define following parameters,

$$\nu = \frac{1}{\bar{\nu}} = 1 \Big/ \left(1 + \frac{z_D}{z_n}\right), \quad \frac{1}{\bar{z}} = \frac{1}{2}\left(\frac{1}{z_n} + \frac{1}{z_D}\right) = \frac{1}{2\nu z_D}. \tag{3}$$

3. Solution of the Diffusion Equation

Assuming a position of the primary cosmic-ray source is given by $\boldsymbol{r}_0(r_0, z_0)$ in the cylindrical coordinate, the diffusion equation has a form

$$[\nabla \cdot D(\boldsymbol{r})\nabla - n(\boldsymbol{r}) v \sigma_p]\, \Phi_p(\boldsymbol{r};\boldsymbol{r}_0) = -\delta(\boldsymbol{r} - \boldsymbol{r}_0)/2\pi r_0, \tag{4}$$

$$\text{with} \quad \Phi_p(r, \pm\infty; r_0, z_0) = 0, \qquad \Phi_p(\infty, z; r_0, z_0) = 0, \tag{5}$$

where v is the particle velocity, and σ_p the inelastic collision cross section with nuclei of the interstellar gas.

Integrating over $\boldsymbol{r}_0(r_0, z_0)$ for $\Phi_p(\boldsymbol{r};\boldsymbol{r}_0)$ with the weight of the source distribution $Q(\boldsymbol{r}_0)$, we get the number density of the cosmic ray at $\boldsymbol{r}$:

$$N_p(\boldsymbol{r}) \;=\; \iint_{-\infty}^{+\infty} Q(\boldsymbol{r}_0)\, \Phi_p(\boldsymbol{r};\boldsymbol{r}_0) d\boldsymbol{r}_0, \tag{6}$$

and the explicit form of the solution at the *Galactic plane* including the rigidity dependence is given by

$$N_p(r;R) = \frac{Q_r}{D_r} \frac{\mathcal{I}_\nu(U_{r,R}) \bar{z}^2 R^{-\beta}}{U_{r,R} I_{\nu-1}(U_{r,R})}, \quad \text{with} \;\; \beta = \gamma + \alpha, \tag{7}$$

where

$$U_{r,R} = 2\nu R^{-\alpha/2} \sqrt{\frac{\sigma_p}{\sigma_r}}, \quad \text{with} \;\; \sigma_r = \frac{D_r}{n_r c z_D^2}, \tag{8}$$

$$\mathcal{I}_\nu(a) = \int_0^1 t^\omega I_\nu(at)dt, \quad \text{with} \;\; \omega = 2\nu/\nu_* - \nu - 1, \tag{9}$$

where ν_* is given by replacing z_n into z_Q in ν defined by Eq. (3). I_ν, $I_{\nu-1}$ are the modified Bessel function with index of ν and $\nu - 1$ respectively, and D_r, n_r and Q_r are given by Eqs. (1a)-(1c) with z=0, respectively.

For typical values, D_0=10^{28}cm^2/sec, n_0=1cm^{-3}, and z_D=1Kpc, appearing in σ_r at the Galactic center (r=0), we have σ_0=34.94mb, which is comparable with the inelastic cross section of proton with the interstellar gas. One should note also $N_p(r;R) \propto R^{-\beta}$ for $R \to \infty$ in Eq. (7).

4. Result and Discussion

In Fig. 1, we demonstrate the energy spectra of typical primary components[5], where the vertical axis is multiplied by $E_P^{2.5}$ (E_P: kinetic energy per particle). We can not conclude which set of (γ, α) appearing in Eq. (2) is the best in this figure alone, namely both sets, for instance (2.4∼2.5, 1/3) and (2.2∼2.3, 1/2), are consistent with the data as long as β $(=\gamma+\alpha)$ is fixed to 2.7-2.8, where $\alpha = 1/3$ corresponds to a Kolmogorov-type of the turbulence, and $\alpha = 1/2$ to a Kraichnan-type.

Now, we would like to stress that there are clear bending points in Fig. 1, somewhere around 10 GeV for proton, while ∼1000GeV/particle for iron, increasing with mass number A. The bending energy/particle, E_B, is obtained easily by solving $d[R^b N_p]/dR = 0$ with use of Eq. (7), ($b = 2.5$ in Fig. 1)

$$E_B \simeq E_{B,0}\Big[1 + \bar{\omega}^2 R_{B,0}^{-\alpha} \frac{2\alpha}{\beta - b} \frac{\sigma_p}{\sigma_r}\Big]. \tag{10}$$

Here σ_r is defined by Eq. (8), and we introduced following variables

$$\frac{E_{B,0}}{AM_u} = \frac{2b - \beta}{\beta - b}, \quad \text{with} \;\; E_{B,0} \simeq ZR_{B,0}, \tag{11}$$

and

$$\bar{\omega} = \sqrt{\frac{1+\nu_\star}{1+\bar{\nu}}}, \quad \text{with} \;\; \frac{1}{\nu_\star} = \frac{1}{\nu} + \frac{1}{\nu_*}, \tag{12}$$

where M_u is the atomic mass unit (=931.5MeV), and A, Z are the mass number and the atomic number of the primary element respectively. For instance, $E_{B,0}/A$=9.3GeV/nucleon for β=2.73, b=2.5, consistent with Fig. 1 in the first order approximation.

One finds also that the bending energy in the second order approximation increases gradually with larger cross section σ_p in Eq. (10), which is approximately proportional to $A^{2/3}$. Based on the preliminary analysis with use of additional data such as 2-ry/1-ry ratio[6], ^{10}Be/^{9}Be[7], diffused-γ[8], we find r_n≈r_Q≈15Kpc,

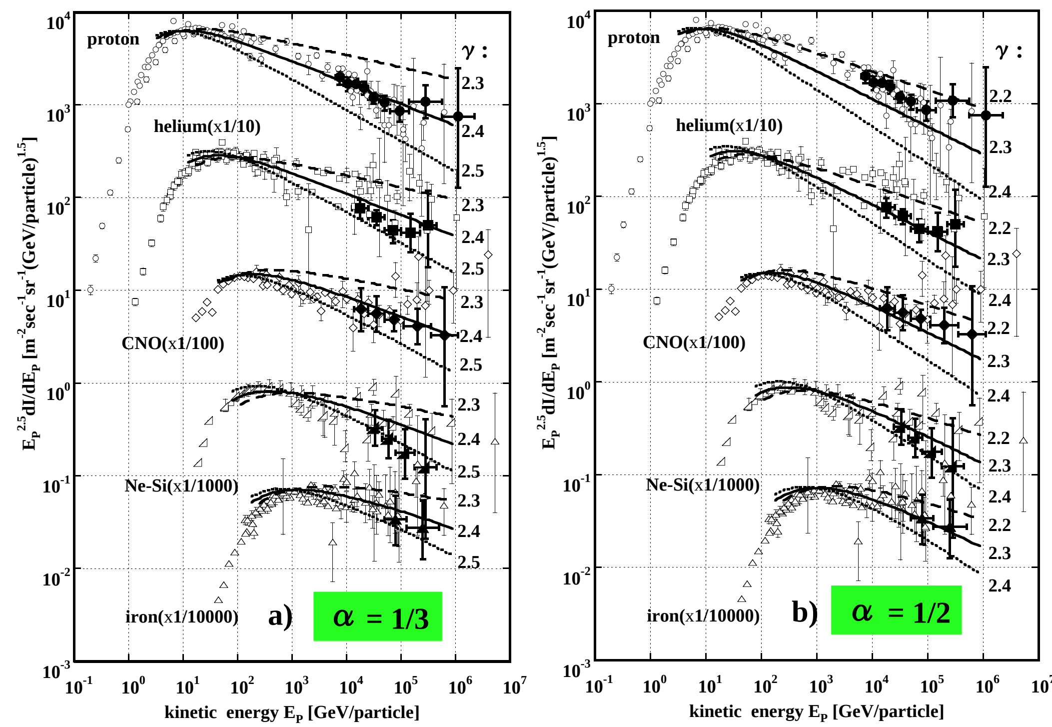

Fig. 1. Energy spectra for individual elements (filled symbols: RUNJOB data[9]).

$(z_D, z_n, z_Q) \approx (1.2,\ 0.5,\ 0.15)$Kpc, $n_0 \approx 1.2\text{cm}^{-3}$, and $D_0 \approx 2.0 \cdot 10^{28}\text{cm}^2/\text{sec}$ at 1GV, while another choice is not yet exculded in this stage. With use of these numerical values, we find E_B=9.9GeV for proton, and 1180GeV/particle for iron, giving a quite consistent result with Fig. 1.

In the present paper, we focused on the high energy region only, and will report the result for the low energy region around 1 GeV/n, where the reacceleration process and ionization loss becomes effective.

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