Dissipation of Hydromagnetic Waves on Energetic Particles: Impact on Interstellar Turbulence and Cosmic Ray Transport

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Abstract

The diffusion of galactic cosmic rays (CR) is considered. It is anticipated that the nonlinear MHD cascade sets the power-law spectrum of turbulence from the principle scale 100 pc to much smaller scales. We found that the dissipation of waves due to the resonant interaction with energetic particles may terminate the cascade at less than 10^{13} cm. The shape of CR diffusion coefficient that was found may explain the observed peaks of secondary to primary nuclei ratios at a few GeV/n.

1. Introduction

The galactic CR have high energy density and they can not always be treated as test particles moving in given magnetic fields. In particular, the stochastic acceleration of CR by MHD waves is accompanied by the damping of waves. The wave damping causes the change of the wave spectrum that in turn affects the particle transport. Thus in principle the study of CR diffusion may need a selfconsistent consideration. We shall see that the CR action on interstellar turbulence should be taken into account at energies below 10 GeV/n. The implementation of this effect in a full scale numerical simulations of CR propagation in the Galaxy was fulfilled by Moskalenko et al. (this Conference).

2. Equations for Cosmic Rays and Interstellar Turbulence

The steady state transport equation that describes the CR diffusion in the interstellar medium is of the form, e.g. [1]:

$$-\nabla D \nabla \Psi - \frac{\partial}{\partial p} p^2 K \frac{\partial}{\partial p} p^{-2} \Psi = q, \tag{1}$$

where $\Psi(p, \mathbf{r}, t)$ is the particle distribution function on momentum normalized on CR number density as $N_{cr}(p) = \int_p^\infty dp\Psi$, $D(p, \mathbf{r})$ is the spatial diffusion coefficient,

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$K(p,\mathbf{r})$ is the diffusion coefficient on momentum, and $q(p,\mathbf{r})$ is the source term. If needed, the supplementary terms which describe the particle energy losses, nuclear fragmentation, and radioactive decays can be added to eq. (1). It is useful to introduce the diffusion mean free path $l = 3D/v$.

It is assumed that the particle diffusion is due to the resonant wave-particle interaction. The diffusion mean free path is determined by the equation [1]: $l = r_g B^2 \left(4\pi k_{res} W(k_{res})\right)^{-1}$, where v is the particle velocity, $r_g = pc/(ZeB)$ is the particle Larmor radius in the average magnetic field B, $k_{res} = 1/r_g$ is the resonant wave number, and $W(k)$ is the spectral energy density of waves defined as $\int dkW(k) = \delta B^2/4\pi$ (δB is the random magnetic field, $\delta B \ll B$). The CR diffusion coefficient has the scaling $D \propto v(p/Z)^a$ for the power law spectrum $W(k) \propto 1/k^{2-a}$. The diffusion on momentum is roughly described by the formula $K = p^2 V_a^2 \left(a(4-a)(4-a^2)D\right)^{-1}$, where V_a is the Alfven velocity.

In spite of the great progress in magnetic hydrodynamics, we do not yet have the well developed theoretical description of interstellar turbulence that would allow one to calculate $W(k)$ in different astrophysical conditions, see review [3]. It is quite possible that there are two almost independent nonlinear cascades of waves in the magnetized plasma where the thermal pressure approximately equals the magnetic field pressure, e.g. [2]. The cascade of Alfven waves (and the slow magnetosonic waves) leads to the Kolmogorov spectrum $W(k) \propto k^{-5/3}$, and the cascade of fast magnetosonic waves leads to the Kraichnan spectrum $W(k) \propto k^{-3/2}$ in the inertial range of wave numbers where the dissipation is absent. Below we consider the interaction of CR with these two cascades separately.

A. Kolmogorov type cascade. In its simplified form, the steady state equations for waves with a nonlinear transfer in k-space can be written as

$$\frac{\partial}{\partial k}\left(C_A k^2 \sqrt{kW(k)\,(4\pi\rho)^{-1}}\,W(k)\right) = -2\Gamma_{cr}(k)W(k) + S_A\delta(k-k_L). \qquad (2)$$

$k \geq k_L$, ρ is the gas density (see [7,8], the original theory of Kolmogorov (1941) was developed for the incompressible liquid without magnetic field). The l.h.s. of eq. (3) describes the nonlinear cascade from small k to large k, C_A is a constant, and approximately equal to 0.3 according to the simulations [10]. The r.h.s. of eq. (3) includes the wave damping on CR and the source term, which works on the main scale $1/k_L = 100$ pc and describes the generation of turbulence by supernova bursts, stellar winds, and superbubbles expansion. In the limit of negligible damping $\Gamma_{cr} = 0$, the solution of eq. (2) gives the Kolmogorov scaling $W(k) \propto k^{-5/3}$.

The equation for wave amplitude attenuation on CR is [1]:

$$\Gamma_{cr}(k) = \pi e^2 V_a^2 \left(2kc^2\right)^{-1} \int_{p_{res(k)}}^{\infty} dp\, p^{-1}\Psi(p), \qquad (3)$$

where $p_{res}(k) = ZeB/ck$. The solution of eqs (2), (3) allows finding the wave spectrum and the determination of the mean free path, which is

$$l(p) = l_{Ko}(p)\left[1 - 2\pi^{3/2}V_a p^{1/3} l_{Ko}^{1/2}(p)\left(3C_A B^2 r_g^{1/2}\right)^{-1}\int_p^{p_L} dp_2 p_2^{2/3}\int_{p_2}^{\infty}\frac{dp_1}{p_1}\Psi(p_1)\right]^{-2}. \tag{4}$$

Here l_{Ko} is the diffusion mean free path calculated for a Kolmogorov spectrum without regard of wave damping, and $p_L = p_{res}(k_L)$. The second term in brackets describes the modification of the mean free path due to the damping of short waves.

B. Iroshnikov-Kraichnan cascade. The simplified equation for waves reads simular to eq. (3) but with the different l.h.s.: $\frac{\partial}{\partial k}\left(C_M k^3 (\rho V_a)^{-1} W^2(k)\right)$, where approximately $C_M = 1$. At $\Gamma_{cr} = 0$, this gives the spectrum $W(k) \propto k^{-3/2}$ first found in [4,6]. Using the same procedure as in the case A, one can obtain:

$$l(p) = l_{Kr}(p)\left[1 - \pi V_a p^{1/2} l_{Kr}(p)\left(2C_M B^2 r_g\right)^{-1}\int_p^{p_L} dp_2 p_2^{1/2}\int_{p_2}^{\infty}\frac{dp_1}{p_1}\Psi(p_1)\right]^{-1}. \tag{5}$$

Here l_{Kr} is the diffusion mean free path calculated for a power law Kraichnan spectrum without regard of wave damping on CR.

As the most abundant species, the CR protons mainly determines the wave dissipation. Their distribution function $\Psi(p)$ should be used to calculate $l(p)$ by the simultaneous solution of eqs (1) and (4) in the case A, and eqs (1) and (5) in the case B. The diffusion mean free path for other nuclei is $l(p/Z)$.

Let us estimate the effect of wave damping at 1 GeV where approximately $l = 1$ pc. The CR energy density is 1 eV/cm^3, $V_a = 10$ km/s, $B = 3$ μG. The second term in brackets in eq.(4) equals 5×10^{-2}, whereas the second term in brackets in eq.(5) equals 10. We conclude that the Kolmogorov type cascade is not much affected by the damping on CR. The Iroshnikov-Kraichnan cascade is significantly affected, and this should lead to the modification of CR transport at energies less than about 10 GeV.

3. Simple Selfconsistent Solution

To demonstrate the effect of wave damping, we consider a simple case of one-dimensional diffusion with the source distribution $q = q_0(p)\delta(z)$, $q_0 \propto p^{-\gamma_S}$ (that corresponds to the infinitely thin disk of CR sources located at $z = 0$) and the flat CR halo of height H, see [5]. Let us assume that stochastic reacceleration does not essentially change the CR spectrum during the time of CR exit from the Galaxy, i.e. one can set $K = 0$ in eq. (1). The solution of eq. (1) in the galactic disk is then $\Psi(p) = 3q_0(p)H\left(2vl(p)\right)^{-1}$. We consider the Iroshnikov-Kraichnan cascade and simplify eq. (5) using the approximation $\int_{p_2}^{\infty} dp_1 p_1^{-1}\Psi(p_1) = \Psi(p_2)/(\gamma_s + 0.5)$. This allows one to find the self consistent diffusion mean free path for protons:

$$l(p) = l_{Kr}(p) \exp\left(3\sqrt{\pi} eH \left(8(\gamma_s + 0.5)C_M\sqrt{\rho}c\right)^{-1} \int_p^{p_L} dp_1 q_0(p_1) v^{-1}(p_1)\right). \quad (6)$$

The form of eq. (6) is close to that needed to explain the peaks in ratios of secondary to primary nuclei in CR at a few GeV/n. Note, that the escape length (the grammage) that determines the production of secondaries is equal to $X = 3\mu H/(2l)$, where μ is the surface gas density of the galactic disk [5].

4. Conclusion

The damping on CR terminates slow Iroshnikov-Kraichnan cascade but probably has no impact on the Kolmogorov-type cascade in the interstellar medium. The estimates were made for the turbulence level that provides the empirical value of CR diffusion coefficient. This finding offers a new explanation of the peaks in the secondary/primary ratios at a few GeV/n: the amplitude of short waves is small because of the damping and thus the low energy particles rapidly exit the Galaxy and almost do not produce secondaries. Another explanation, e.g. [9], that the peaks are produced by CR reacceleration ($K \neq 0$) on an undisturbed Kolmogorov-type spectrum remains as a viable alternative. The quantitative analysis of this problem is presented by Moskalenko et al. at this Conference.

Many aspects of this topic including the structure of turbulence at the main scale (the theory predicts that $S_A \gg S_M$), the anisotropy in k-space (the waves propagates at large angles to local magnetic field in the Alfvenic turbulence and this should increase the diffusion coefficient compared to (2)), and the consequences for the interpretation of data on interstellar turbulence remain beyond the scope of the present short paper.

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5. References

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