New Constraints On The Nature Of Space-Time Planck Scale Fluctuations Using X-ray and TeV Gamma-Ray Observations

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Abstract

The space-time metric is widely believed to be subject to stochastic fluctuations induced by quantum gravity at the Planck scale. This paper describes a work based on two different phenomenological approaches to this topic. By interpreting the ideas developed in these two approaches in the light of each other, it is shown that the constraints on the nature of Planck scale space-time fluctuations already set by the observation of very high energy electrons and gamma-rays are much stronger than have been shown so far. It appears that for the kind of Planck scale fluctuations implied by several models, including the most naive one, to be consistent with the observations, the transformation laws between different reference frames must be modified in order to allow the Planck scale to be observer-independent.

1. First approach: the effects of Planck scale space-time fluctuations on kinematics

A general way of describing the space-time fluctuations at the Planck scale is to write $\sigma_x/x = f(x_P/x)$, where x stands for the length l or the time t, with $f \ll 1$ for $x \gg x_P$ and $f \gtrsim 1$ for $x \lesssim x_P$. In this case, $f(x)$ can be approximated with the lower order term of its expansion in the range $x \gg x_P$ in the following way [18]:

$$\frac{\sigma_x}{x} \simeq a_0 \left(\frac{x_P}{x}\right)^{\alpha} \tag{1}$$

where both α and a_0 are positive constants of order 1. The naive choice for α is 1, in which case $\sigma_x \simeq x_P \,\forall\, x$. This choice is also the first order term given by quantum loop gravity (see [18]). Other models of quantum space-time give alternative values such as $\alpha = 1/2$ (random-walk scenario) or $\alpha = 2/3$ (holographic principle of Wheeler and Hawking). This is discussed in [18,21] and references therein.

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Let us calculate the effects of space-time fluctuations on particles' momentum and energy. It is assumed that the de Broglie wavelengths of the particles follow the fluctuations of space-time (assumption 1) and that the four components of these fluctuations are uncorrelated (assumption 2).

Expanding eq.1 to the first order in E/E_P in the usual dispersion relation, in the energy range $m^2 \ll E^2 \simeq p^2 \ll E_P^2$, one obtains:

$$m^2 = E^2 - p^2 + \frac{\eta E^{\alpha+2}}{E_P^{\alpha}} \tag{2}$$

where η is distributed as a gaussian with $\mu = 0$ and $\sigma = 2\sqrt{2}a_0$. A similar expression is obtained in [2,3], for the case $\alpha = 1$.

The choice of the reference frame in which to apply eqs.1 and 2 raises an important issue concerning special relativity: in which reference frames do the fluctuations have the Planck scale, if they exist ? Let's consider the three following cases:

- **case A:** Planck scale space-time fluctuations do not exist.
- **case B:** if the fluctuations have the Planck scale in all reference frames, then the laws of coordinate transformations between different inertial reference frames would have to depart from pure Lorentz transformations to let this scale be invariant. This is the milestone of Doubly Special Relativity (DSR) theories, in which both the velocity of light and the Planck scale of length and mass are observer-independent scales [5,7,8,9,19].
- **case C:** on the other hand, space-time fluctuations may have the Planck scale in one preferred reference frame only, and boosted values of this scale in other reference frames. Indeed, there is a preferred reference frame in the Universe: the one where the Cosmic Microwave Background (CMB) appears isotropic. This case has been considered in many phenomenological studies of the effects that a fluctuating space-time would have on kinematics [14], although it implies the abandonment of the relativity principle which stipulates that laws of physics should be the same for all inertial observers.

A consequence of eq.2 is that at each measurement, the measured values of E and p are different from their mean values. Consider interactions where the energy exchanged by the particles involved is $\ll E_P$. The typical scales of length and time of these interactions are much larger that the Planck ones. Hence, one should stipulate independent fluctuations for each initial and final particle [3].

The effects of Planck scale space-time fluctuations on kinematics would have a threshold of $\simeq$ 15 TeV in case C, if $\alpha = 1$, and an even lower one if $\alpha < 1$. Let us introduce a second approach to highlight two of these effects which are particularly relevant to the derivation of experimental constraints.

2. Introduction of the second approach and outcome

The studies described in [14,15], [10,11] and [13] are based on an equation similar to eq.2, where η is a constant instead of a fluctuating term. It has been shown in [14,15] that if η could take negative values*, photons and electrons above 15 TeV would undergo 1 vertex interactions. Photons would decay into e^+e^- pairs and electrons would radiate spontaneously, hence neither would be observed. This result can be generalized for $\alpha \leq 1$.

Such high energy particles have been observed, respectively by HEGRA [2] and CANGAROO [23] and by ASCA X-ray observations of the Crab nebula [16]. Hence, it can be concluded that $\eta < 0$ and $\alpha \leq 1$ are not consistent with the observations.

If η takes stochastic values as shown in the first approach, then negative values are possible. If such values are ruled out, then any stochastic behaviour allowing $\eta < 0$ is ruled out as well. Therefore, case C is ruled out, provided the assumptions made so far are valid. The demonstration of this result is given with more details in [17].

3. Conclusion

The conclusion of this paper is that Planck scale space-time fluctuations described by an exponent $\alpha \leq 1$ are consistent with the observations only if the Planck scale is observer-independent, in the framework of the assumptions made here. Concerned models are: the naive description of space-time, as described by $\sigma_x \simeq x_P \; \forall x$ [3,4] and implied by quantum loop gravity, the random walk scenario, and the holographic principle of Wheeler and Hawking (see [18]). As a result, there are two possibilites. The first one is that Planck scale space-time fluctuations do not exist. The second one is that if they exist according to one of the above models the Planck scale has to be observer-independent, which implies that the laws of coordinate transformations between different inertial reference frames have to be changed. If on the other hand both possibilites were ruled out by other means, one could show that at least one of the assumptions made here is wrong, or that $\alpha > 1$. This would also bring a significant clue concerning the development of quantum gravity theories.

A similar conclusion has been reached independently and very recently in [4], using the same approach as in this paper. Reference [18] is also a recent study of Planck scale space-time fluctuations, which is based on stellar interferometry. It is cited several times in this paper, regarding the developments made in its beginning. It concludes that space-time does not fluctuate at the Planck scale.

*The reader who wishes to read [14,15] should note that η is defined here with the opposite sign as in these references.

However, the demonstration it uses is controversial [12,20]. Laser interferometers which will be used in gravitational wave detectors like VIRGO and LIGO will also be able to detect Planck scale space-time fluctuations if they exist [6], or rule them out, giving one more independent insight on the nature of space-time at the Planck scale.

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4. References

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