Non-extensivity Parameter in Thermodynamical Model of Hadronic Interactions

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Abstract

In this paper we used the non-extensive thermodynamical picture of hadronic interaction to examine measured transverse momenta distributions for wide interaction energy range. We determined model parameters and their energy dependencies: one of the parameters V (hadronization volume) scales with the particle multiplicity, another T (temperature in the Boltzmann limit) is constant and the most important non-extensivity parameter q varies smoothly with energy.

1. Introduction

The thermodynamical treatment of hadronization was recently successfully used by [1] to describe average multiplicities in $p\bar{p}$, e^+e^-, AA high energy interactions. However, the main problem of such treatment: non-exponential tails of transverse momentum distributions remain. One way of describing them was proposed in [2] by introducing the pre-hadronization dynamics of fireballs. Different solution is to generalize the standard, Boltzmann statistics following Tsallis idea [3]. It was shown that such non-extensive statistics works well for $p_\perp$ spectra in e^+e^- [4], and could be used for $p\bar{p}$ interactions too [5]. Here we would like to show that the results of detailed calculations with the Tsallis generalization of Becattini canonical thermodynamics. We found that this is remarkably good way of describing particle production in $p\bar{p}$ reactions in the whole experimentally available energy range.

2. Calculations

The main idea of thermodynamical approach is the partition function.

$$Z\left(Q^0\right) \;=\; \sum_{states} \prod_{j,k} \exp\left(-\frac{n_{jk}\,E_{jk}}{T}\right)\,\delta_{Q,Q^0}\;, \tag{1}$$

where Q^0 is the fireball quantum state (and it is conserved in our approach), j denotes particle species and n is the respective k-th momentum cell occupation

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number. Non-extensive generalization of the exponential Boltzmann factor is:

$$\exp\left(-\frac{\sqrt{p^2+m_j^2}}{T}\right) \quad\longrightarrow\quad \left(1+\frac{(q-1)}{T}\sqrt{p^2+m_j^2}\right)^{-\frac{q}{q-1}} \quad\equiv\; x_j\ , \qquad (2)$$

where q is called the non-extensivity parameter. The partition function is all what is needed to obtain any property of the system. The transverse momentum distribution is given as

$$f\left(p_T^2\right)\mathrm{d}p_T^2 = \sum_j\sum_{n=1}^{\infty}(\pm 1)^{n+1}\gamma^{n\,s_j}\,(2J_j+1)\frac{V}{2\pi^2}\int \mathrm{d}p_L\; x_j^n\,\frac{Z(Q^0-nq_j)}{Z(Q^0)}\,\mathrm{d}p_T^2\ , \quad (3)$$

where γ is a common strangeness suppression factor, J_j is the spin of the j-th type hadron (of all quantum numbers given by the vector q_j), $\pm$ differs bosons from fermions, and V denotes the hadronization volume.

We performed all numerical integrations in Eq.(3) taking into account almost hundred species of daughter particles. All unstable particles were then decayed. In calculations we took into consideration all decays with branching ratios down to 1%. For all three non-extensive model parameters V, T and q the $p_\perp$ distributions were tabulated in steps of 10 fm^3, 10 MeV and 0.01, respectively, and they were used further to fit the data.

3. Non-extensive description of experimental data

We have used the (invariant) $p_\perp$ distributions measured by different experiments in the energy range from 100 GeV/c laboratory momentum up to $\sqrt{s}$ of 1800 GeV [6]. For each distribution we tried to find the set of non-extensive thermodynamic parameters (V, T and q) describing in the best way the high $p_\perp$ tail (it was found that fits work very well starting already from 0.5 GeV). The dependence on hadronization volume V is (as expected for not so small values) only the normalization factor, so in fact only T and q were fitted. The T parameter is sensitive to smaller $p_\perp$'s while the q describes the non-exponential tails. No significant and systematic energy dependence of T were found, and eventually we found that this parameter can be fixed at 130 MeV. For each data set the value of non-extensivity the value of q was determined. The quality of fits was very good (quite close to this shown in the figures below).

Then we tried to check, if we can describe better the data with not a single value of q for each energy but with the assumption that q has a distribution with non-zero width. We tried to convolute distributions for single q with Gaussian (and also exponential functions of higher power polynomials of q, but with no significant improvement). We enhanced small $p_\perp$'s allowing one additional Boltzmann $p_\perp$ distribution with its temperature as a free parameter to be adjusted. Some examples are shown in the Fig.1. Dotted lines (indistinguishable for high

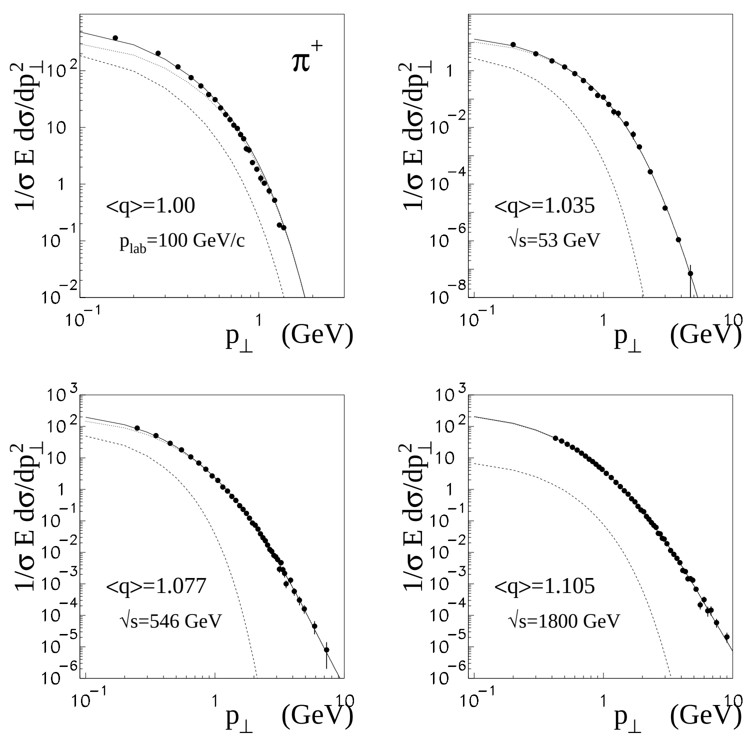

Fig. 1. Transverse momentum distributions and their non-extensive fits (see text).

$p_\perp$'s from the solid ones) represents non-extensive component, the low $p_\perp$ Boltzmann is shown as dashed lines, and solid lines represents the sum. Mean values of q are also given. The width of the q distribution for 100 GeV/c was found to be 0, and with the $\langle q\rangle = 1$ it corresponds to exact Boltzmann $p_\perp$ distribution (the temperature fixed at 130 MeV do not match exactly, so in fact, there are two Boltzmann distributions there). Starting from the ISR energies the deviations from $q = 1$ starts to appear as well as some theirs non-zero spreads. However, distributions of q are very narrow. Their widths (Gaussian σ's) were found to be of about $0.005 - 0.01$. Such narrow distributions (and absence of significant deviations of their shapes from simply Gaussian) support one of our conclusions that the single q value non-extensive statistical description of the hadronization process is close to the reality, no evidence suggesting its further extension is seen in the $p_\perp$ data. Another fact supporting this statement is the smooth q parameter energy dependence. The low energy Boltzmann limit (seen in our 100 GeV/c plot in Fig.1) $q = 1$ is well known. Together with the upper (asymptotic, infinite energy) limit of 1.25 [3] it can be combine in

$$q(E) = \Big[\, 1.25 \;-\; A\, s^{-B} \,\Big]_{\geq 1} \qquad (4)$$

and in the Fig.3. the fit of Eq.(4) to obtained $\langle q\rangle$ values is presented. The best

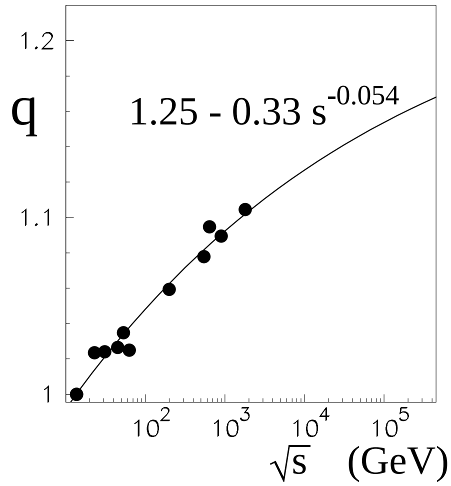

Fig. 2. Energy dependence of the non-extensivity parameter.

values of A and B are found to be 0.33 and 0.054, respectively

4. Summary

We have shown that the non-extensive thermodynamical picture of hadron production provide very good description of transverse momentum distributions for all examined experimental spectra[6]. The T parameter was found to be constant and equal to 130 MeV and we found smooth and simple energy dependence of non-extensivity parameter q.

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