Energy Fluctuation of Tau Leptons Emerging from Earth

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Abstract

Tau leptons can be produced by tau neutrinos interact inside Earth via charged current interaction. The tau lepton energy could be quite different from tau neutrino energy. A Monte-Carlo simulation is developed to study the effects of inelasticity and energy loss tau lepton. The combined energy fluctuation is less than 50% at the neutrino energy less than $10^{17}$ eV, then it rises sharply to 100% at approximately $10^{18}$ eV. The best energy range for detection of Earth-skimming neutrinos is $10^{15}$ eV to $10^{18}$ eV.

1. Introduction

At energy above PeV ($10^{15}$ eV), detecting neutrinos skimming through the Earth is more efficient than neutrinos from atmosphere [1], especially for tau neutrino. The tau neutrino interacts with nucleon and produce tau lepton via charged current interaction, $\nu_\tau + X \rightarrow \tau + X'$. Observation of tau emerging from mountain is not only proof of the existence of high energy neutrinos, but also a $\tau$ appearance experiment to support $\nu_\mu$ to $\nu_\tau$ oscillation.

When tau leptons travel through the Earth (standard rock in this study), they loss some energy and some taus decay before they escape the dense target. The energy of emerging tau ($E_\tau$) differs from initial neutrino energy ($E_\nu$). To determine the best energy range for detection of Earth-skimming neutrinos, a Monte-Carlo simulation is developed to study the effects of inelasticity and energy loss. We select two targets, 10 km and 100 km of rocks, to represent the neutrino passing through mountain or skimming through Earth.

2. Conversion from Tau neutrino to Tau lepton

The charged current (CC) cross-section of neutrino-nucleon interaction is calculated in a way similar to Gandhi et al. [2] but use the latest parton distribution function CTEQ6 [3]. The total cross-sections, shown in left side of Fig. 1, are approximately 38% higher than that of Gandhi et al. [2] at 10 PeV.
The interaction length of the neutrino in target of density $\rho$ is $\lambda_\nu = 1/(\sigma N A \rho)$.

The differential cross-sections $d\sigma/dy$ for several energies are shown in right side of Fig. 1, where the inelasticity $y$ is the fraction of energy carried away by interacting nucleon, $y = 1 - (E_\tau/E_\nu)$. For each $\nu_\tau \to \tau$ interaction, inelasticity $y$ is sampled randomly from a probability function $(d\sigma(E_\nu)/dy)/\sigma$.

![Fig. 1.](image.png)

Fig. 1. The left figure shows the total cross-section $\sigma$ based on CTEQ6 parton distribution (red solid line, [this work]) and CTEQ4 (blue dash line, [2]). The right figure shows the differential cross-sections based on CTEQ6.

3. Propagation of tau in materials

The energy loss of tau in rock is dominated by photonuclear interaction at the energy above $10^{14}$ eV, others effects include bremsstrahlung, pair production and ionization [4]. These energy loss mechanisms are modeled by continuous energy loss, $-dE/dX = \alpha + \beta E = \beta' E$, where $\beta' \equiv \alpha/E + \beta$.

The tau decay length is $d_\tau = E c_\tau \tau/m_\tau \approx 49.03 \times (E/{\text{PeV}})$ m, where $m_\tau$ and $\tau_\tau$ are mass and life time of $\tau$ and $E$ is in PeV. For a small step of slant depth $dX$, $\beta'$ can be assumed as constant, such that

$$E_f = E_i \exp\left\{-\beta'(E_i)E_i dX\right\},$$

and the survival probability is

$$P_s(E_f) = P_s(E_i) \exp\left\{-\frac{m_\tau}{c_\tau \tau_\tau \beta'} \left(\frac{1}{E_i} - \frac{1}{E_f}\right)\right\},$$

The total survival probability $P_s$ and the final energy $E_f$ for tau pass through certain depth in target is calculated by an iterative method until tau lepton exits target. Then $P_s$ is compared with a random probability $P$; if $P_s > P$, then this tau lepton can survive the journey and accepted as one event. The range of tau $R_\tau$ is defined by $\int X P_s(X) dX/ \int P_s(X) dX$. 

\[\int X P_s(X) dX/ \int P_s(X) dX.\]
4. Simulation and Results

The neutrinos are injected with a series of constant energy \( E_\nu \); for each \( E_\nu \), \( 10^4 \) accepted tau leptons are generated. The number of neutrinos \( N_\nu(E_\nu) \) and the number of tau leptons \( N_\tau(E_\nu, E_\tau) \) are recorded, then the conversion matrix \( C \) can be obtained by

\[
\phi_\tau(E_\tau) = C(E_\nu, E_\tau)\Phi_\nu(E_\nu)
\]

where \( \Phi_\nu(E_\nu) \) is the flux of neutrinos at energy \( E_\nu \).

This efficiency is defined by the average number of tau leptons produced from one neutrino.

\[
\epsilon(E_\nu) = \sum_{E_\tau} \frac{N_\tau(E_\nu, E_\tau)}{N_\nu(E_\nu)}
\]

Fig. 2 shows the matrix \( N_\tau(E_\nu, E_\tau) \) for conversion in 10 km of rock. The conversion efficiency is defined by the average number of tau leptons produced from one neutrino.

\[
\epsilon(E_\nu) \approx \frac{R_\tau}{\lambda_\nu} [1].
\]

This efficiency is approximately \( \epsilon(E_\nu) \approx R_\tau/\lambda_\nu \) [1]. Fig. 2 shows this conversion efficiency. At energy below 30 PeV, \( \epsilon(E_\nu) \propto E^{1.5} \). The spectral index 1.5 comes from \( R_\tau \approx d_\tau \propto E \) and \( 1/\lambda_\nu \propto \sigma \propto E^{0.36} \) [2]. However, at higher energy, the energy loss reduce the tau range significantly, \( \epsilon(E_\nu) \) gradually decrease to \( \propto E^{-3} \).

5. Energy fluctuation

When the thickness of target is larger than \( R_\tau \), on average, tau are produced with mean energy \( (1 - \bar{y})E_\nu \), where \( \bar{y} \) is mean inelasticity; then tau pass through distance \( R_\tau \). However, the larger the \( y \), the smaller the survival probability \( P_s \). Therefore, target become a filter to select smaller \( y \). The \( \bar{y} \) in targets is less than that in vacuum. When the thickness of target is less than \( R_\tau \), then \( \bar{y} \) will
have smaller difference than that in vacuum. The mean distance tau traveled is only one half of the thickness of target. Therefore the mean values of $E_\tau$ depend on the thickness of target, shown on the left side of Fig. 3.

Shown in the right side of Fig. 3, tau energy fluctuation $\sigma E_\tau/E_\tau$ is less than 30% at $E_\nu < 100$ PeV and rise sharply at higher energy. Fluctuation of inelasticity $\sigma y$ stays at approximately 20%. At $E_\nu > 100$ PeV, $\sigma E_\tau/E_\tau$ are dominated by energy loss. The longer the $R_\tau$, the larger the energy fluctuation $\sigma E_\tau/E_\tau$.

The other two sources for fluctuation of energy are the different tau decay channels and air shower. Assuming the fluctuation from those two terms are 20% and 30% respectively, then the combined energy resolutions will be approximately 50% at $E_\nu < 10^{17}$ eV and then rise to over 100% at $E_\nu > 3 \times 10^{18}$.

![Fig. 3.](image)

**Fig. 3.** The left figure show the fraction of mean $E_\tau$ and $E_\nu$. The right figures show the relative fluctuation of $E_\tau$.

6. Conclusions

From the conversion efficiency, shown in Fig. 2, the Earth-skimming event is more efficient than atmospheric events at above $10^{15}$ eV. In order to have good energy resolution $< 100\%$, the upper limit is around $10^{18}$ eV. Although the energy resolution degraded at $E_\nu > 10^{18}$ eV, the high conversion efficiency is still appearing for detection of ultra high energy neutrinos.

7. References

3. Pumplin J. et al., 2002, JHEP, 0207, 012