Radio Pulses Generated by Showers in Different Dense Media

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Abstract

The frequency and angular dependences of radio pulses originated by high energy showers in different dense media is compared. Simulations made with two Monte Carlo packages, ZHS and GEANT4, give consistent results. Analytical expressions are inferred to characterize the radio pulse spectrum up to very high frequencies.

1. Introduction

Ultra high energy neutrino detection (UHE\(\nu\)) is one of the main experimental challenges in astroparticle physics. The search for coherent Cherenkov radio pulses in the MHz-GHz frequency range produced by neutrino induced showers in transparent, dense media, provides an alternative to the conventional methods of detecting UHE\(\nu\)es. Electrons produced in charged-current \(\nu\) interactions originate electromagnetic showers that develop an excess of negative charge mainly due to Compton scattering. When the wavelength of the emitted radiation is larger than the typical dimensions of the shower the emission is coherent, and the power in radio waves scales as the square of shower energy. These predictions were confirmed in a recent accelerator experiment at SLAC that used sand as detector medium [5]. Several experiments that exploit this technique are in progress, such as the RICE experiment [4], an array of antennas buried in the polar ice cap, ANITA which will use an antenna in a balloon to search for pulses from Antarctica [3] and the GLUE experiment [2] which uses the visible side of the Moon as target for UHE\(\nu\) and cosmic ray interactions.

The aim of this work is to compare numerical predictions of the frequency spectrum of the radio signal emitted by showers in different dense media. The scaling of the radiopulse with medium properties is studied and related to the properties of the shower development. Understanding these relations is crucial for the design and interpretation of current and future experiments.
2. The Calculation of the Radio Pulse

The Fourier time-transform in the Fraunhofer limit of the electric field produced by a charge \( q \) moving at constant velocity \( \vec{v} \) for a small time interval \((t_1, t_1 + \delta t)\) from the position \( \vec{r}_1 \) is given by:

\[
R \vec{E}(\omega, \vec{x}) = \frac{1}{2\pi \epsilon_0} \left( \frac{q \mu_r i \omega}{c^2} \right) e^{ikR \epsilon_i \omega (t_1 - \frac{\vec{n} \cdot \vec{r}_1}{c})} \vec{v}_\perp \delta t
\]

\( \epsilon_0 (c) \) is the permittivity (speed of light) in vacuum, \( \mu_r (n) \) is the relative magnetic permeability (the refraction index) of the medium, \( \omega \) the angular frequency, \( k = \frac{\omega}{c} \), and, \( \vec{v}_\perp \) the projection of \( \vec{v} \) in the direction perpendicular to the direction of observation \( (\vec{n}) \). Eq. 1 is an approximation valid at the Cherenkov angle \( (\theta_c) \) for all frequencies and at sufficiently low frequencies \( (\omega < \delta t^{-1}) \) for all angles. It is the algorithm behind the calculations presented in this work. In order to calculate the electric field produced by a shower, the contributions from each particle track, as given by Eq. 1, have to be summed. To do this numerically the tracks must be subdivided so that \( \vec{v} \) is approximately constant and \( \omega \delta t \) is sufficiently small for all frequencies of interest.

Fig. 1. shows the frequency spectrum of the electric field from 1 TeV showers in ice as calculated with GEANT 4 and the ZHS [6] Monte Carlo packages. The electric field spectrum rises linearly with frequency up to a turnover point \( (\nu_c(\theta)) \) which depends on the observation angle. The absolute normalization of the field in the region \( \nu < \nu_c \) is determined by the distance travelled by the excess charge in the direction perpendicular to the direction of observation i.e., on
the excess projected tracklength. The differences in electric field normalization are $\sim 3\%$ at frequencies below $\sim 500$ MHz [1]. These are due to differences in tracklength between both simulations. At higher frequencies the remaining discrepancies can be attributed to different algorithms for subdividing the particle tracks before applying Eq. [1]. This point will be further explored in the future.

For observation at the Cherenkov angle, different stages in the longitudinal development of the shower contribute with the same phase. The emission is expected to be coherent until the wavelength is of the order of the lateral spread of the shower. Away from $\theta_c$, the electric field spectrum rises linearly with frequency until the wavelength of observation is a fraction of the longitudinal size of the shower. Using a simple model of a shower in which the charge excess travels at the speed of light along a length proportional to $X_0$, and has a width proportional to $R_M$, it is easy to show that $\nu_c(\theta_c) \propto [R_M \tan \theta_c]^{-1}$, whereas at $\theta \neq \theta_c$ the relation becomes $\nu_c(\theta) \propto [nX_0(\cos \theta_c - \cos \theta)]^{-1}$. The effective length of the shower grows as the observation angle separates from $\theta_c$ and exceeds the lateral spread. As a result the turnover frequencies are typically smaller at $\theta \neq \theta_c$.

3. The Spectrum in Different Media

We have also performed simulations in salt and lunar rock. The results obtained confirm that the general properties of the electric field spectra can be obtained from simple considerations about shower development and the properties of radio waves in the medium. Given the considerations above, the relevant parameters are the excess projected tracklength, the radiation length ($X_0$), the Molière radius ($R_M$ characterizing the transverse spread of the shower) and the
Table 1. Some properties of the materials considered in this work. We also show the cutoff frequencies at $\theta_c$ and at 90 deg., as well as the value of the electric field $(R \times E)$ at 10 MHz for observation at $\theta_c$ as predicted in ZHS for an average of twenty 1 TeV showers in those materials.

<table>
<thead>
<tr>
<th>Medium</th>
<th>$\rho$ [g/cm$^3$]</th>
<th>$R_M$ [cm]</th>
<th>$X_0$ [cm]</th>
<th>$\theta_c$ [deg]</th>
<th>$\nu_c (\theta_c)$ [GHz]</th>
<th>$\nu_c (90$ deg$)$ [MHz]</th>
<th>$R \times E$ [V/MHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice</td>
<td>0.92</td>
<td>11.2</td>
<td>39.05</td>
<td>55.8</td>
<td>$\sim$ 1.5</td>
<td>$\sim$ 25.0</td>
<td>2.0 $10^{-7}$</td>
</tr>
<tr>
<td>Salt</td>
<td>2.05</td>
<td>5.9</td>
<td>10.81</td>
<td>64.8</td>
<td>$\sim$ 2.0</td>
<td>$\sim$ 100.0</td>
<td>8.4 $10^{-8}$</td>
</tr>
<tr>
<td>Moon</td>
<td>3.00</td>
<td>3.9</td>
<td>7.53</td>
<td>56.3</td>
<td>$\sim$ 5.0</td>
<td>$\sim$ 125.0</td>
<td>5.5 $10^{-8}$</td>
</tr>
</tbody>
</table>

Cherenkov angle.

The excess projected tracklength relates to energy loss and is approximately the same in different media when expressed in g cm$^{-2}$. There is a weak dependence on $Z$, the mean atomic number of the medium, which makes very small differences for the media discussed here and we neglect it. As a result the magnitude of the electric field can be considered to scale with the inverse of the medium density, i.e. as $\rho^{-1} \sin \theta$. Once the normalization is fixed the turnover frequencies can be obtained from the longitudinal and transverse characteristic lengths of the showers, which scale respectively with $X_0$ and $R_M$.

Fig. 2. shows the electric field spectrum from a single 100 TeV photon shower in salt, ice and lunar rock as obtained using ZHS. These new versions of ZHS agree with GEANT simulations. In Table 1. we show $\rho$, $R_M$, $X_0$ and $\theta_c$ for these media, the turnover frequencies at $\theta_c$ and 90 deg., as well as the normalization of the electric field at 10 MHz when observing at $\theta_c$. It is straightforward to check that the scaling relations discussed in Sec. 2 work at a level better than 10% for both the normalizations of the electric fields and the turnover frequencies.

In conclusion, we have shown that the main features of the electric field spectrum in different media scale rather well with medium parameters. The scaling allows for instance the prediction of radiopulse properties in different media without performing an explicit Monte Carlo simulation.

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