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## Some Unsettled Questions in the Problem of Neutrino Oscillations. Mechanisms of Neutrino Oscillations

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Beshtoev Kh. M.

*Joint Inst. for Nucl. Research., Joliot Curie 6, 141980 Dubna, Moscow region, Russia and Inst. Appl. Math. and Autom. KBSRC of RAS, Nalchik, Russia*

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### Abstract

In the modern theory of neutrino oscillations constructed in the framework of the theory particle physics there appears three types of neutrino transitions (oscillations). Then, in order to solve the question of which type of neutrino transitions (oscillations) are realized in nature, in experiments, it is necessary to study profile of neutrino transitions in dependence on distances for determination lengths and angle mixings. At present it is presumed that Dirac and Majorana neutrino oscillations can be realized. It is shown that we cannot put Majorana neutrinos in the standard weak interactions theory without violation of the gauge invariance. Also is shown that the mechanism of resonance enhancement of neutrino oscillations in matter cannot be realized without violation of the law of energy-momentum conservation. Then, it is obvious that there can be only realized transitions (oscillations) between Dirac neutrinos with different flavors.

### 1. Introduction

In previous works [1,2] it was shown that there are three types of vacuum neutrino oscillations. One of them is the standard mechanism of neutrino oscillations, [3] where angle of neutrino mixings is defined by neutrino mass differences and nondiagonal mass terms; and in other cases the angle of mixings is maximal ( $\pi/4$ ). Let us come to critical consideration of mechanisms of neutrino oscillations.

### 2. Impossibility of resonance enhancement of neutrino oscillations in matter

In three different approaches - by using mass Lagrangian [4-6], by using the Dirac equation [5, 6], and using the operator formalism [7] - the author of this work has discussed the problem of mass generation in the standard weak interactions, and came to a conclusion that the standard weak interaction cannot generate masses of fermions since the right-handed components of fermions do not participate in these interactions. Also it is shown [8] that the equation for Green function of the weak-interacting fermions (neutrinos) in the matter coincides with the equation for Green function of fermions in vacuum, and the law of conservation of the energy and the momentum of neutrino in matter will be fulfilled [7] only if the energy  $W$  of polarization of matter by the neutrino or the corresponding term in Wolfenstein equation, is zero (it means that neutrinos cannot generate permanent polarization of matter). These results lead to the conclusion: resonance enhancement of neutrino oscillations in matter does not exist.

The simplest method to prove the absence of the resonance enhancement of neutrino oscillations in matter is:

If we put an electrical (or strong) charged particle  $a$  in matter, there would arise polarization of matter. Since the field around particle  $a$  is spherically symmetrical, the polarization must also be spherically symmetrical. Then, the particle will be left at rest and the law of energy and momentum conservation is fulfilled.

If we put a weakly interacting particle  $b$  (a neutrino) in matter, then, since the field around the particle has a left-right asymmetry (weak interactions are left interactions with respect to the spin direction [9]), polarization of matter would have to be nonsymmetrical, i.e. on the left side there arises maximal polarization and on the right one there is zero polarization. Since polarization of the matter is asymmetrical, there arises asymmetrical interaction of the particle (the neutrino) with matter, and the particle cannot be at rest and will be accelerated. Then, the law of energy momentum conservation will be violated. The only way to fulfill the law of energy and momentum conservation is to demand that polarization of matter be absent in the weak interactions. The same situation will take place in vacuum. It is also necessary to remark that the Super-Kamiokande datum on day-night asymmetry [10] is

$$A = (D - N) / \left(\frac{1}{2}(D + N)\right) = -0.021 \pm 0.020(stat) + 0.013(-0.012)(syst). \quad (1)$$

and there remains no hope on possibility of the resonance enhancement of neutrino oscillations in matter. It means that the forward scattering amplitude of the weak interactions has a specific behavior.

### 3. Majorana Neutrino Oscillations

At present, it is supposed [3, 11] that the neutrino oscillations could be connected with Majorana neutrino oscillations. I will show that we cannot put Majorana neutrinos in the standard Dirac theory. It means that on experiments the Majorana neutrino oscillations cannot be observed.

Majorana fermion in Dirac representation has the following form [3, 12]:

$$\chi^M = \frac{1}{2}[\Psi(x) + \eta_C \Psi^C(x)], \quad \Psi^C(x) \rightarrow \eta_C C \bar{\Psi}^T(x), \quad (2)$$

where  $\eta_C$  is a phase,  $C$  is a charge conjunction,  $T$  is a transposition.

From Exp. (2) we see that Majorana fermion  $\chi^M$  has two spin projections  $\pm\frac{1}{2}$  and then the Majorana spinor can be rewritten in the following form:

$$\chi^M(x) = \begin{pmatrix} \chi_{+\frac{1}{2}}(x) \\ \chi_{-\frac{1}{2}}(x) \end{pmatrix}. \quad (3)$$

The mass Lagrangian of Majorana neutrinos in the case of two neutrinos  $\chi_e, \chi_\mu$  ( $-\frac{1}{2}$  components of Majorana neutrinos, and  $\bar{\chi}_{\dots}$ , is the same as Majorana fermion with the opposite spin projection) in the common case has the following form:

$$\mathcal{L}'_M = -\frac{1}{2}(\bar{\chi}_e, \bar{\chi}_\mu) \begin{pmatrix} m_{\chi_e} & m_{\chi_e \chi_\mu} \\ m_{\chi_\mu \chi_e} & m_{\chi_\mu} \end{pmatrix} \begin{pmatrix} \chi_e \\ \chi_\mu \end{pmatrix}. \quad (4)$$

By diagonalizing this mass matrix by standard methods, one obtains the following expression:

$$\mathcal{L}'_M = -\frac{1}{2}(\bar{\nu}_1, \bar{\nu}_2) \begin{pmatrix} m_{\nu_1} & 0 \\ 0 & m_{\nu_2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}, \quad \begin{aligned} \nu_1 &= \cos\theta\chi_e - \sin\theta\chi_\mu \\ \nu_2 &= \sin\theta\chi_e + \cos\theta\chi_\mu \end{aligned}. \quad (5)$$

These neutrino oscillations are described by standard expressions (see [1-3]).

The standard theory of weak interactions is constructed on the base of local gauge invariance of Dirac fermions. In this case Dirac fermions have the following lepton numbers  $l_l$ , which are conserved,  $l_l, l = e, \mu, \tau$ , and Dirac antiparticles have lepton numbers with the opposite sign  $\bar{l} = -l_l$ .

Gauge transformation of Majorana fermions can be written in the form:

$$\chi'_{+\frac{1}{2}}(x) = \exp(-i\beta)\chi_{+\frac{1}{2}}(x), \quad \chi'_{-\frac{1}{2}}(x) = \exp(+i\beta)\chi_{-\frac{1}{2}}(x). \quad (6)$$

Then lepton numbers of Majorana fermions are  $l^M = \sum_i l_i^M(+1/2) = -\sum_i l_i^M(-1/2)$ , i. e., antiparticle of Majorana fermion is the same fermion with the opposite spin projection.

Now we come to discussion of the problem of the place of Majorana fermion in the standard theory of weak interactions [13].

To construct the standard theory of weak interactions, [9] Dirac fermions are used. The absence of contradiction of this theory with the experimental data confirms that all fermions are Dirac particles.

Now, if we want to put the Majorana fermions into the standard theory we must take into account that, in the common case, the gauge charges of the Dirac and Majorana fermions are different (especially well it is seen in the example of Dirac fermion having an electrical charge since it cannot have a Majorana charge (it is worth to remind that in the weak currents the fermions are included in the couples form)). In this case we cannot just include Majorana fermions in the standard theory of weak interactions by gauge invariance manner. Then, in the standard theory the Majorana fermions cannot appear.

If we include Majorana neutrinos into the standard theory, then in experiments we must see the following reactions:  $\chi_l + A(Z) \rightarrow l^- + A(Z + 1)$  with probability 1/2 and  $\chi_l + A(Z) \rightarrow l^+ + A(Z - 1)$  with the same probability (where  $x = e, \mu, \tau$ ), since Majorana neutrinos are superpositions of Dirac neutrinos and antineutrinos. Obviously, all the available experimental data [14] does not confirm this predictions, therefore we cannot consider this mechanism as realistic one for neutrino oscillations.

#### 4. Vacuum Transitions (Oscillations) of Flavour Neutrinos (Conclusions)

In the work [15] Maki et al. supposed that there could exist transitions between flavour neutrinos  $\nu_e, \nu_\mu$ . Afterwards,  $\nu_\tau$  was found and then  $\nu_e, \nu_\mu, \nu_\tau$  transitions could be possible.

In spite of that, the published works on neutrino oscillations considered only one type of neutrino oscillations (the 1-st type is the type considered in ref. [3]); nevertheless, we cannot suppose that this problem is solved. In order to solve this problem it is necessary to know the precise data about neutrino oscillations, i. e. to research neutrino oscillations at different distances for determination of lengths and mixing angles.

What conclusion can we do from the available experimental data?

1. The Super-Kamiokande experiment [16] on atmospheric neutrinos has detected the deficit of muonic neutrinos. The analysis shows that they can transit only in  $\nu_\tau$  neutrinos. The  $\nu_\mu \rightarrow \nu_e$  transition in this experiment is not observed. From this fact we can conclude (taking into account SNO results) that the length of  $\nu_\mu \rightarrow \nu_\tau$  transitions is of the order of the Earth diameter, and the angle  $\theta$  of  $\nu_\mu \rightarrow \nu_\tau$  transitions is near to the maximal mixing angle  $\theta \cong \pi/4$ . Then, the

length of  $\nu_\mu \rightarrow \nu_e$  transitions is much more than the Earth diameter. The SNO experimental data also confirms - through neutral current registration -  $\nu_\mu \rightarrow \nu_\tau$  transitions with the same mixing angle.

2. Using the SNO experimental data [17] on straight registration neutrinos (by neutral and charge currents in case  $\nu_e$  and neutral current in the case  $\nu_\mu, \nu_\tau$ ) we can come to the following conclusion: the primary  $\nu_e$  neutrinos transit in approximately equal proportions in  $\mu_e, \nu_\mu, \nu_\tau$  neutrinos, i. e., mixing angles  $\theta_{(\dots)}$  of  $\nu_e, \nu_\mu, \nu_\tau$  are approximately equal to the maximal angles of mixing. The length of  $\nu_e \rightarrow \nu_\mu, \nu_\tau$  oscillations is less than the distance to the Sun. Obviously, for more detailed analysis of this problem it is necessary to obtain more precise data on neutrino oscillations (i. e. to study profile of neutrino oscillations in dependency of distances).

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