A Modern theory of neutrino oscillations

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1. Introduction

In the old theory of neutrino oscillations [1, 2], constructed in the framework of the quantum mechanics in analogy with the theory of $K^o, \bar{K}^o$ oscillation, it was supposed that mass eigenstates are ν_1, ν_2, ν_3 neutrino states, but not physical neutrino states ν_e, ν_μ, ν_τ; and that the neutrinos ν_e, ν_μ, ν_τ were created as superpositions of ν_1, ν_2, ν_3 states. This meant that the ν_e, ν_μ, ν_τ neutrinos have no definite mass, i.e. their masses may vary in dependence on the ν_1, ν_2, ν_3 admixture in the ν_e, ν_μ, ν_τ states. Then if, for example, the electron neutrino transits into muon (tau) neutrino and then this muon ν_ν (ν_τ) neutrino is decayed on electron neutrino plus something, and as result of this oscillation we get energy from vacuum which equals the mass difference $\Delta E \sim m_{\nu_\mu} - m_{\nu_e}$. Then again, this electron neutrino transits into muon (tau) neutrino, which is decayed again, and we get energy and etc. (**So we have perpetuum mobile!**). Obviously, in this process the law of energy-momentum conservation cannot be fulfilled. Probably, the only way to restore the law of energy-momentum conservation is to demand that this process is virtually one. Then, these oscillations will be virtual ones and they will be described in the framework of the uncertainty relations. Besides, every particle must be created on its mass shell and will be left on its mass shell while passing through vacuum. Let us pass to consideration of this approach.

2. The Theory of Neutrino Oscillations

In the modern theory on neutrino oscillations [3,4], constructed in the framework of the particle physics theory, it is supposed that:

1) The physical states of the ν_e, ν_μ, ν_τ neutrinos are eigenstates of the weak interaction and, naturally, the mass matrix of ν_e, ν_μ, ν_τ neutrinos is diagonal. All available experimental results indicate that the lepton numbers l_e, l_μ, l_τ are well conserved, i.e. the standard weak interactions do not violate the lepton numbers.

2) Then, in order to violate the lepton numbers, it is necessary to introduce an interaction violating these numbers. It is equivalent to introducing nondiagonal mass terms in the mass matrix of ν_e, ν_μ, ν_τ. By diagonalizing this matrix, we go to the ν_1, ν_2, ν_3 neutrino states. Exactly like it was in the case of K^o mesons created in strong interactions, when mainly $K^o, \bar{K}^o$ mesons were produced, in the considered case ν_e, ν_μ, ν_τ, but not ν_1, ν_2, ν_3, neutrino states are mainly created in the weak interactions (this is so, because the contribution of the lepton numbers violating interactions in this process is too small). And in such case no oscillations take place.

3) Then, when the ν_e, ν_μ, ν_τ neutrinos are passing through vacuum, they will be converted into superpositions of the ν_1, ν_2, ν_3 owing to the presence of the interactions violating the lepton number of neutrinos, and will be left on their mass shells. And then, oscillations of the ν_e, ν_μ, ν_τ neutrinos will take place according to the standard scheme [2-4]. Whether these oscillations are real or virtual, it will be determined by the masses of the physical neutrinos ν_e, ν_μ, ν_τ.

i) If the masses of the ν_e, ν_μ, ν_τ neutrinos are equal, then the real oscillation

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of the neutrinos will take place.

ii) If the masses of the ν_e, ν_μ, ν_τ are not equal, then the virtual oscillation of the neutrinos will take place. To make these oscillations real, these neutrinos must participate in the quasielastic interactions, in order to undergo transition to the mass shell of the other appropriate neutrinos by analogy with $\gamma - \rho^o$ transition in the vector meson dominance model.

3. Neutrino Oscillation Types

The mass matrix of ν_e and ν_μ neutrinos has diagonal form. Due to the presence of the interaction violating the lepton numbers, a nondiagonal term appears in this matrix and then this mass matrix is transformed into a nondiagonal matrix (CP is conserved), then the lagrangian of mass of the neutrinos takes the following form ($\nu \equiv \nu_L$):

$$\begin{array}{c} \mathcal{L}_M = -\frac{1}{2}\left[m_{\nu_e}\bar{\nu}_e\nu_e + m_{\nu_\mu}\bar{\nu}_\mu\nu_\mu + m_{\nu_e\nu_\mu}(\bar{\nu}_e\nu_\mu + \bar{\nu}_\mu\nu_e)\right] \equiv \\ \equiv -\frac{1}{2}(\bar{\nu}_e, \bar{\nu}_\mu)\left(\begin{array}{cc} m_{\nu_e} & m_{\nu_e\nu_\mu} \\ m_{\nu_\mu\nu_e} & m_{\nu_\mu} \end{array}\right)\left(\begin{array}{c} \nu_e \\ \nu_\mu \end{array}\right) \end{array}, \tag{1}$$

which is diagonalized by turning through the angle θ and (see ref. in [2]) and then this lagrangian (1) transforms into the following one:

$$\mathcal{L}_M = -\frac{1}{2}\left[m_1\bar{\nu}_1\nu_1 + m_2\bar{\nu}_2\nu_2\right], \tag{2}$$

$$where \quad m_{1,2} = \frac{1}{2}\left[(m_{\nu_e} + m_{\nu_\mu}) \pm \left((m_{\nu_e} - m_{\nu_\mu})^2 + 4m^2_{\nu_\mu\nu_e}\right)^{1/2}\right],$$

and angle θ is determined by the following expression:

$$tg2\theta = \frac{2m_{\nu_e\nu_\mu}}{(m_{\nu_\mu} - m_{\nu_e})}, \qquad \begin{array}{c} \nu_e = cos\theta\nu_1 + sin\theta\nu_2, \\ \nu_\mu = -sin\theta\nu_1 + cos\theta\nu_2. \end{array} \tag{3}$$

From eq.(3) one can see that if $m_{\nu_e} = m_{\nu_\mu}$, then the mixing angle is equal to $\pi/4$ independently of the value of $m_{\nu_e\nu_\mu}$:

$$sin^2 2\theta = \frac{(2m_{\nu_e\nu_\mu})^2}{(m_{\nu_e} - m_{\nu_\mu})^2 + (2m_{\nu_e\nu_\mu})^2}, \left(\begin{array}{cc} m_{\nu_1} & 0 \\ 0 & m_{\nu_2} \end{array}\right). \tag{4}$$

It is interesting to remark that expression (4) can be obtained from the Breit-Wigner distribution [5]

$$P \sim \frac{(\Gamma/2)^2}{(E - E_0)^2 + (\Gamma/2)^2}, \tag{5}$$

by using the following substitutions:

$$E = m_{\nu_e}, \;\; E_0 = m_{\nu_\mu}, \;\; \Gamma/2 = 2m_{\nu_e,\nu_\mu}, \tag{6}$$

where $\Gamma/2 \equiv W(...)$ is a width of $\nu_e \rightarrow \nu_\mu$ transition, then we can use a standard method [4, 6] for calculating this value.

The expression for time evolution of ν_1, ν_2 neutrinos (see (2), (4)) with masses m_1 and m_2 is

$$\nu_1(t) = e^{-iE_1 t}\nu_1(0), \quad \nu_2(t) = e^{-iE_2 t}\nu_2(0), \quad where \quad E_k^2 = (p^2 + m_k^2), k = 1, 2. \tag{7}$$

If neutrinos are propagating without interactions, then

$$\nu_e(t) = cos\theta e^{-iE_1 t}\nu_1(0) + sin\theta e^{-iE_2 t}\nu_2(0), \nu_\mu(t) = -sin\theta e^{-iE_1 t}\nu_1(0) + cos\theta e^{-iE_2 t}\nu_2(0). \tag{8}$$

Using the expression for ν_1 and ν_2 from (7), and putting it into (8), one can get expressions for $\nu_e(t)$ and $\nu_\mu(t)$. The probability that neutrino ν_e created at the time $t = 0$ will be transformed into ν_μ at the time t is an absolute value of amplitude $\nu_\mu(0)$ in (8) squared, i. e.

$$P(\nu_e \to \nu_\mu) = \mid (\nu_\mu(0) \cdot \nu_e(t)) \mid^2 = \frac{1}{2}\sin^2 2\theta \left[1 - cos((m_2^2 - m_1^2)/2p)t\right], \tag{9}$$

where it is supposed that $p \gg m_1, m_2; E_k \simeq p + m_k^2/2p$.

The expression (9) presents the probability of neutrino aroma oscillations. The angle θ (mixing angle) characterizes value of mixing. The probability $P(\nu_e \to \nu_\mu)$ is a periodical function of distances, where the period is determined by the following expression:

$$L_o = 2\pi \frac{2p}{\mid m_2^2 - m_1^2 \mid}. \tag{10}$$

And probability $P(\nu_e \to \nu_e)$ that the neutrino ν_e created at time $t = 0$ is preserved as ν_e neutrino at time t, is given by the absolute value of the amplitude of $\nu_e(0)$ in (8) squared. Since the states in (8) are normalized states, then $P(\nu_e \to \nu_e) + P(\nu_e \to \nu_\mu) = 1$.

So, we see that aromatic oscillations caused by nondiagonality of the neutrinos mass matrix violate the law of the $-\ell_e$ and ℓ_μ lepton number conservations. However, in this case, as one can see from the above exp. , the full lepton numbers $\ell = \ell_e + \ell_\mu$ are conserved.

We can also see that there are two cases of ν_e, ν_μ transitions (oscillations) [4,6].

1. If we consider the transition of ν_e into ν_μ particle, then

$$sin^2 2\beta \cong \frac{4m^2_{\nu_e,\nu_\mu}}{(m_{\nu_e} - m_{\nu_\mu})^2 + 4m^2_{\nu_e,\nu_\mu}}, \tag{11}$$

How can we understand this $\nu_e \to \nu_\mu$ transition?

If $2m_{\nu_e,\nu_\mu} = \frac{\Gamma}{2}$ is not zero, then it means that the mean mass of ν_e particle is m_{ν_e} and this mass is distributed by $sin^2 2\beta$ (or by the Breit-Wigner formula), and the probability of the $\nu_e \to \nu_\mu$ transition differs from zero and it is defined by masses of ν_e and ν_μ particles and m_{ν_e,ν_μ}, which is computed in the framework of the standard method, as pointed out above.

So, this is a solution of the problem of the origin of mixing angle in the theory of vacuum oscillations.

In this case, the probability of $\nu_e \to \nu_\mu$ transition (oscillation) is described by the following expression:

$$P(\nu_e \to \nu_\mu, t) = sin^2 2\beta sin^2 \left[\pi t \frac{\mid m^2_{\nu_1} - m^2_{\nu_2} \mid}{2p_{\nu_e}}\right], \tag{12}$$

where p_{ν_e} is a momentum of ν_e particle.

2. If we consider the virtual transition of ν_e into ν_μ neutrino at $m_{\nu_e} = m_{\nu_\mu}$ (i.e. without changing the mass shell), then $tg 2\beta = \infty, \beta = \pi/4, \quad and \quad sin^2 2\beta = 1$.

In this case the probability of the $\nu_e \to \nu_\mu$ transition (oscillation) is described by the following expression:

$$P(\nu_e \to \nu_\mu, t) = \left[\pi t \frac{4m^2_{\nu_e,\nu_\mu}}{2p_a}\right]. \tag{13}$$

In order to make these virtual oscillations real, their participation in quasielastic interactions is necessary for the transitions to their own mass shells [6]. It is clear that the $\nu_e \to \nu_\mu$ transition is a dynamical process.

3. The third type of transitions (oscillations?) can be realized by mixings of the fields (neutrinos) in analogy with the vector dominance model ($\gamma - \rho^o$ and $Z^o - \gamma$ mixings) in a way it takes place in the particle physics. Since the weak couple constants $g_{\nu_e}, g_{\nu_\mu}, g_{\nu_\tau}$ of ν_e, ν_μ, ν_τ neutrinos are nearly equal in reality, i.e. $g_{\nu_e} \simeq g_{\nu_\mu} \simeq g_{\nu_\tau}$, then the angle mixings are nearly maximal:

$$sin\theta_{\nu_e \nu_\mu} \simeq \frac{g_{\nu_e}}{\sqrt{g^2_{\nu_e} + g^2_{\nu_\mu}}} = \frac{1}{\sqrt{2}} \simeq sin\theta_{\nu_e \nu_\tau} \simeq sin\theta_{\nu_\mu \nu_\tau}. \tag{14}$$

Therefore, if the masses of these neutrinos are equal (which is hardly probable), then transitions between neutrinos will be real; and if the masses of these neutrinos are not equal, then transitions between neutrinos will be virtual in analogy with $\gamma - \rho^o$ transitions.

Conclusion

The question is: which type of neutrino oscillations (transitions) is realized in the Nature?

References

1. Gribov V., Pontecorvo B.M., Phys. Lett. B, 1969, vol.28, p.493.
2. Bilenky S.M., Pontecorvo B.M., Phys. Rep., C41(1978)225; Boehm F., Vogel P., Physics of Massive Neutrinos: Cambridge Univ. Press, 1987, p.27, p.121; Bilenky S.M., et al., Rev. of Mod. Phys., 1977, v.59, p.631.
3. Beshtoev Kh.M., JINR Commun. E2-92-318, Dubna, 1992; JINR Rapid Communications, N3[71]-95.
4. Beshtoev Kh.M., Internet Pub. hep-ph/9911513; The Hadronic Journal, v.23, 2000, p.477; Proceedings of 27th Intern. Cosmic Ray Conf., Germany, Hamburg, 7-15 August 2001, v.3, p. 1186.
5. Blatt J.M., Waiscopff V.F., The Theory of Nuclear Reactions, INR T.R. 42.
6 . Beshtoev Kh.M., JINR Commun. E2-99-307, Dubna, 1999; JINR Commun. E2-99-306, Dubna, 1999; Beshtoev Kh.M., Phys. of Elem. Part. and Atomic Nucl. (Particles and Nuclei), 1996, v.27, p.53.