1. Introduction

In the old theory of neutrino oscillations [1, 2], constructed in the framework of the quantum mechanics in analogy with the theory of $K^0, \bar{K}^0$ oscillation, it was supposed that mass eigenstates are $\nu_1, \nu_2, \nu_3$ neutrino states, but not physical neutrino states $\nu_e, \nu_\mu, \nu_\tau$; and that the neutrinos $\nu_e, \nu_\mu, \nu_\tau$ were created as superpositions of $\nu_1, \nu_2, \nu_3$ states. This meant that the $\nu_e, \nu_\mu, \nu_\tau$ neutrinos have no definite mass, i.e. their masses may vary in dependence on the $\nu_1, \nu_2, \nu_3$ admixture in the $\nu_e, \nu_\mu, \nu_\tau$ states. Then if, for example, the electron neutrino transits into muon (tau) neutrino and then this muon $\nu_\mu$ ($\nu_\tau$) neutrino is decayed on electron neutrino plus something, and as result of this oscillation we get energy from vacuum which equals the mass difference $\Delta E \sim m_{\nu_\mu} - m_{\nu_e}$. Then again, this electron neutrino transits into muon (tau) neutrino, which is decayed again, and we get energy and etc. (So we have perpetuum mobile!). Obviously, in this process the law of energy-momentum conservation cannot be fulfilled. Probably, the only way to restore the law of energy-momentum conservation is to demand that this process is virtually one. Then, these oscillations will be virtual ones and they will be described in the framework of the uncertainty relations. Besides, every particle must be created on its mass shell and will be left on its mass shell while passing through vacuum. Let us pass to consideration of this approach.

2. The Theory of Neutrino Oscillations

In the modern theory on neutrino oscillations [3,4], constructed in the framework of the particle physics theory, it is supposed that:

1) The physical states of the $\nu_e, \nu_\mu, \nu_\tau$ neutrinos are eigenstates of the weak interaction and, naturally, the mass matrix of $\nu_e, \nu_\mu, \nu_\tau$ neutrinos is diagonal. All available experimental results indicate that the lepton numbers $l_e, l_\mu, l_\tau$ are well conserved, i.e. the standard weak interactions do not violate the lepton numbers.

2) Then, in order to violate the lepton numbers, it is necessary to introduce an interaction violating these numbers. It is equivalent to introducing nondiagonal mass terms in the mass matrix of $\nu_e, \nu_\mu, \nu_\tau$. By diagonalizing this matrix, we go to the $\nu_1, \nu_2, \nu_3$ neutrino states. Exactly like it was in the case of $K^0$ mesons created in strong interactions, when mainly $K^0, \bar{K}^0$ mesons were produced, in the considered case $\nu_e, \nu_\mu, \nu_\tau$, but not $\nu_1, \nu_2, \nu_3$, neutrino states are mainly created in the weak interactions (this is so, because the contribution of the lepton numbers violating interactions in this process is too small). And in such case no oscillations take place.

3) Then, when the $\nu_e, \nu_\mu, \nu_\tau$ neutrinos are passing through vacuum, they will be converted into superpositions of the $\nu_1, \nu_2, \nu_3$ owing to the presence of the interactions violating the lepton number of neutrinos, and will be left on their mass shells. And then, oscillations of the $\nu_e, \nu_\mu, \nu_\tau$ neutrinos will take place according to the standard scheme [2-4]. Whether these oscillations are real or virtual, it will be determined by the masses of the physical neutrinos $\nu_e, \nu_\mu, \nu_\tau$.

i) If the masses of the $\nu_e, \nu_\mu, \nu_\tau$ neutrinos are equal, then the real oscillation
of the neutrinos will take place.

ii) If the masses of the $\nu_e, \nu_\mu, \nu_\tau$ are not equal, then the virtual oscillation of the neutrinos will take place. To make these oscillations real, these neutrinos must participate in the quasielastic interactions, in order to undergo transition to the mass shell of the other appropriate neutrinos by analogy with $\gamma - \rho'$ transition in the vector meson dominance model.

3. Neutrino Oscillation Types

The mass matrix of $\nu_e$ and $\nu_\mu$ neutrinos has diagonal form. Due to the presence of the interaction violating the lepton numbers, a nondiagonal term appears in this matrix and then this mass matrix is transformed into a nondiagonal matrix ($CP$ is conserved), then the lagrangian of mass of the neutrinos takes the following form ($\nu \equiv \nu_L$):

$$\mathcal{L}_M = -\frac{1}{2} \left[ m_{\nu_e} \bar{\nu}_e \nu_e + m_{\nu_\mu} \bar{\nu}_\mu \nu_\mu + m_{\nu_e} \nu_\mu (\bar{\nu}_e \nu_\mu + \bar{\nu}_\mu \nu_e) \right] \equiv -\frac{1}{2} (\bar{\nu}_e, \bar{\nu}_\mu) \begin{pmatrix} m_{\nu_e} & m_{\nu_e} \nu_\mu \\ m_{\nu_\mu} \nu_\mu & m_{\nu_\mu} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \quad (1)$$

which is diagonalized by turning through the angle $\theta$ and (see ref. in [2]) and then this lagrangian (1) transforms into the following one:

$$\mathcal{L}_M = -\frac{1}{2} \left[ m_1 \bar{\nu}_1 \nu_1 + m_2 \bar{\nu}_2 \nu_2 \right], \quad (2)$$

where $m_{1,2} = \frac{1}{2} \left[ \left( m_{\nu_e} + m_{\nu_\mu} \right) \pm \left( (m_{\nu_e} - m_{\nu_\mu})^2 + 4m_{\nu_e \nu_\mu}^2 \right)^{1/2} \right]$, and angle $\theta$ is determined by the following expression:

$$\tan 2\theta = \frac{2m_{\nu_e \nu_\mu}}{m_{\nu_\mu} - m_{\nu_e}}, \quad \nu_e = \cos \theta \nu_1 + \sin \theta \nu_2, \quad \nu_\mu = -\sin \theta \nu_1 + \cos \theta \nu_2. \quad (3)$$

From eq.(3) one can see that if $m_{\nu_e} = m_{\nu_\mu}$, then the mixing angle is equal to $\pi/4$ independently of the value of $m_{\nu_e \nu_\mu}$:

$$\sin^2 2\theta = \frac{\left(2m_{\nu_e \nu_\mu}\right)^2}{\left(m_{\nu_\mu} - m_{\nu_e}\right)^2 + \left(2m_{\nu_e \nu_\mu}\right)^2}, \quad \begin{pmatrix} m_{\nu_1} & 0 \\ 0 & m_{\nu_2} \end{pmatrix}. \quad (4)$$

It is interesting to remark that expression (4) can be obtained from the Breit-Wigner distribution [5]

$$P \sim \frac{(\Gamma/2)^2}{(E - E_0)^2 + (\Gamma/2)^2}, \quad (5)$$

by using the following substitutions:

$$E = m_{\nu_e}, \quad E_0 = m_{\nu_\mu}, \quad \Gamma/2 = 2m_{\nu_e \nu_\mu}. \quad (6)$$

where $\Gamma/2 \equiv W(...) \equiv \text{a width of } \nu_e \rightarrow \nu_\mu \text{ transition}$, then we can use a standard method [4, 6] for calculating this value.
The expression for time evolution of $\nu_1, \nu_2$ neutrinos (see (2), (4)) with masses $m_1$ and $m_2$ is

$$\nu_1(t) = e^{-iE_1t}\nu_1(0), \quad \nu_2(t) = e^{-iE_2t}\nu_2(0), \quad \text{where} \quad E_k^2 = (p_k^2 + m_k^2), \quad k = 1, 2. \quad (7)$$

If neutrinos are propagating without interactions, then

$$\nu_e(t) = \cos \theta e^{-iE_1t}\nu_1(0) + \sin \theta e^{-iE_2t}\nu_2(0), \quad \nu_\mu(t) = -\sin \theta e^{-iE_1t}\nu_1(0) + \cos \theta e^{-iE_2t}\nu_2(0). \quad (8)$$

Using the expression for $\nu_1$ and $\nu_2$ from (7), and putting it into (8), one can get expressions for $\nu_e(t)$ and $\nu_\mu(t)$. The probability that neutrino $\nu_e$ created at the time $t = 0$ will be transformed into $\nu_\mu$ at the time $t$ is an absolute value of amplitude $\nu_\mu(0)$ in (8) squared, i.e.

$$P(\nu_e \rightarrow \nu_\mu) = |(\nu_\mu(0) \cdot \nu_e(t))|^2 = \frac{1}{2} \sin^2 2\theta [1 - \cos((m_2^2 - m_1^2)/2p)t], \quad (9)$$

where it is supposed that $p \gg m_1, m_2; \quad E_k \approx p + m_k^2/2p$.

The expression (9) presents the probability of neutrino aroma oscillations. The angle $\theta$ (mixing angle) characterizes value of mixing. The probability $P(\nu_e \rightarrow \nu_\mu)$ is a periodic function of distances, where the period is determined by the following expression:

$$L_o = 2\pi \frac{2p}{|m_2^2 - m_1^2|}. \quad (10)$$

And probability $P(\nu_e \rightarrow \nu_e)$ that the neutrino $\nu_e$ created at time $t = 0$ is preserved as $\nu_e$ neutrino at time $t$, is given by the absolute value of the amplitude of $\nu_e(0)$ in (8) squared. Since the states in (8) are normalized states, then $P(\nu_e \rightarrow \nu_e) = P(\nu_\mu \rightarrow \nu_e) = 1$.

So, we see that aromatic oscillations caused by nondiagonality of the neutrinos mass matrix violate the law of the $-\ell_e$ and $\ell_\mu$ lepton number conservations. However, in this case, as one can see from the above expression, the full lepton numbers $\ell = \ell_e + \ell_\mu$ are conserved.

We can also see that there are two cases of $\nu_e, \nu_\mu$ transitions (oscillations) [4,6].

1. If we consider the transition of $\nu_e$ into $\nu_\mu$ particle, then

$$\sin^2 2\beta \simeq \frac{4m_{\nu_e,\nu_\mu}^2}{(m_{\nu_e} - m_{\nu_\mu})^2 + 4m_{\nu_e,\nu_\mu}^2}, \quad (11)$$

How can we understand this $\nu_e \rightarrow \nu_\mu$ transition?

If $2m_{\nu_e,\nu_\mu} = \frac{m_{\nu_e}}{2}$ is not zero, then it means that the mean mass of $\nu_e$ particle is $m_{\nu_e}$ and this mass is distributed by $\sin^2 2\beta$ (or by the Breit-Wigner formula), and the probability of the $\nu_e \rightarrow \nu_\mu$ transition differs from zero and it is defined by masses of $\nu_e$ and $\nu_\mu$ particles and $m_{\nu_e,\nu_\mu}$, which is computed in the framework of the standard method, as pointed out above.

So, this is a solution of the problem of the origin of mixing angle in the theory of vacuum oscillations.
In this case, the probability of $\nu_e \rightarrow \nu_\mu$ transition (oscillation) is described by the following expression:

$$P(\nu_e \rightarrow \nu_\mu, t) = \sin^2 2\beta \sin^2 \left[ \pi t \frac{m_{\nu_1}^2 - m_{\nu_2}^2}{2p_{\nu_e}} \right],$$

(12)

where $p_{\nu_e}$ is a momentum of $\nu_e$ particle.

2. If we consider the virtual transition of $\nu_e$ into $\nu_\mu$ neutrino at $m_{\nu_e} = m_{\nu_\mu}$ (i.e. without changing the mass shell), then $\tan 2\beta = 1, \beta = \pi/4, \quad \text{and} \quad \sin^2 2\beta = 1$.

In this case the probability of the $\nu_e \rightarrow \nu_\mu$ transition (oscillation) is described by the following expression:

$$P(\nu_e \rightarrow \nu_\mu, t) = \left[ \pi t \frac{4m_{\nu_e,\nu_\mu}^2}{2p_a} \right].$$

(13)

In order to make these virtual oscillations real, their participation in quasielastic interactions is necessary for the transitions to their own mass shells [6]. It is clear that the $\nu_e \rightarrow \nu_\mu$ transition is a dynamical process.

3. The third type of transitions (oscillations?) can be realized by mixings of the fields (neutrinos) in analogy with the vector dominance model ($\gamma - \rho^0$ and $Z^0 - \gamma$ mixings) in a way it takes place in the particle physics. Since the weak couple constants $g_{\nu_e}, g_{\nu_\mu}, g_{\nu_\tau}$ of $\nu_e, \nu_\mu, \nu_\tau$ neutrinos are nearly equal in reality, i.e. $g_{\nu_e} \simeq g_{\nu_\mu} \simeq g_{\nu_\tau}$, then the angle mixings are nearly maximal:

$$\sin \theta_{\nu_e,\nu_\mu} \simeq \frac{g_{\nu_e}}{\sqrt{g_{\nu_e}^2 + g_{\nu_\mu}^2}} = \frac{1}{\sqrt{2}} \simeq \sin \theta_{\nu_e,\nu_\tau} \simeq \sin \theta_{\nu_\mu,\nu_\tau}.$$  

(14)

Therefore, if the masses of these neutrinos are equal (which is hardly probable), then transitions between neutrinos will be real; and if the masses of these neutrinos are not equal, then transitions between neutrinos will be virtual in analogy with $\gamma - \rho^0$ transitions.

**Conclusion**

The question is: which type of neutrino oscillations (transitions) is realized in the Nature?

**References**