A high-accurate and high-efficient Monte Carlo code by improved Molière functions with ionization

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Abstract

Although the Molière theory of multiple Coulomb scattering is less accurate in tracing solid angles than the Goudsmit and Saunderson theory due to the small angle approximation, it still acts very important roles in developments of high-efficient simulation codes of relativistic charged particles like cosmic-ray particles. Molière expansion is well explained by the physical model, that is the normal distribution attributing to the high-frequent moderate scatterings and subsequent correction terms attributing to the additive large-angle scatterings. Based on these physical concepts, we have improved a high-accurate and high-efficient Monte Carlo code taking account of ionization loss.

1. Introduction

We have confirmed the Molière process of multiple Coulomb scattering is well explained by the splitting model of single scattering cross-section [1,2]. High-frequent moderate scattering less than the splitting angle produces the central gaussian distribution as a first approximation and the low-frequent large-angle scattering larger than the splitting angle gives the far reaching long tail to the angular distribution. We have examined the Molière process by Monte Carlo simulations with Rutherford cross-section and searched for the most effective splitting angle to divide the single scattering into the moderate and the large-angle scatterings.

In case of the splitting angle of $\chi_{\rm B}$ or $e^{B/2}$ times the screening angle $\sqrt{e}\chi_{\rm a}$, corresponding to the Molière expansion, the traversed thickness is not thick enough for the moderate scattering to produce gaussian distribution, so that the distortion from the gaussian distribution due to the higher approximation terms are found in the central distribution. On the other hand in case of the splitting angle of $\chi_{\rm C}$, well known as the threshold angle to give the large-angle scattering once within the traversed thickness, we have confirmed the moderate scatterings produce accurate enough central gaussian distribution with the predicted width in our Monte Carlo investigations. So we have thought it most effective to separate the single scattering at $\chi_{\rm C}$, to replace the resultant deflections from multiple

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moderate scatterings with the gaussian distribution, and to add the large-angle scattering stochastically once within the traversed thickness on average.

For very thin thickness, we cannot apply multiple scattering theory any more as well as the above sampling method, where we apply the single-scattering sampling directly. Smooth transfer between the both samplings has been confirmed in this investigation.

We can apply this method also under the ionization process. The results obtained by the present theory and method is compared with those by the traditional method of fixed energy, where the thickness is divided into many small stepsizes so as particle energies not to be changed much [3].

2. Central distributions produced by high-frequent moderate scatterings

According to the splitting cross-section method [1,2], the Molière angular distribution is reconstructed by the folding integrals between the central distribution produced by moderate scatterings and the k-times large-angle scattering. Although the multiple moderate scatterings below the splitting angle should reach to the gaussian distribution after traverse of thick enough depths, it is not clear which shape the central distribution will make after the traverse of certain finite depths. We have examined the shape of central distribution for splitting angles of χ_{B} and χ_{C}.

Central distribution will be predicted from

$$\frac{d\tilde{f}}{dt} = 2\pi\tilde{f}\int_0^\infty [J_0(\zeta\theta) - 1]\sigma_{\mathrm{M}}(\theta)\theta d\theta. \tag{1}$$

Taking account of the higher-order Fourier components indicated as e.g. (A12) of Scott [4], we have

$$2\pi\tilde{f}_{\mathrm{M}} = \exp[-\frac{\theta_{\mathrm{M}}^2\zeta^2}{4}]\{1 + \frac{1}{4B}(1 - e^{-B})e^{2-2C}(\frac{\theta_{\mathrm{M}}^2\zeta^2}{4})^2 + \cdots\} \tag{2}$$

for splitting angle of χ_{B}, so that

$$2\pi f_{\mathrm{M}}(\vartheta)d\vec{\vartheta} = d\vec{\vartheta}\{f^{(0)}(\vartheta) + \frac{1}{2B}(1 - e^{-B})e^{2-2C}f_2^{(2)}(\vartheta) + \cdots\} \tag{3}$$

with $\vartheta \equiv \theta/(\theta_{\mathrm{G}}\sqrt{B/\Omega})$. Likewise we have

$$2\pi f_{\mathrm{M}}(\vartheta)d\vec{\vartheta} = d\vec{\vartheta}\{f^{(0)}(\vartheta) + \frac{1}{2(\ln n_{\mathrm{R}})^2}(1 - \frac{1}{n_{\mathrm{R}}})f_2^{(2)}(\vartheta) + \cdots\} \tag{4}$$

with $\vartheta \equiv \theta/(\chi_{\mathrm{C}}\sqrt{\ln n_{\mathrm{R}}})$ for splitting angle of χ_{C}. Considerable distortion from the gaussian is seen for splitting at χ_{B} as indicated in Fig. 1, on the other hand good agreements with the gaussian is seen for splitting at χ_{C} as indicated in Fig. 2.

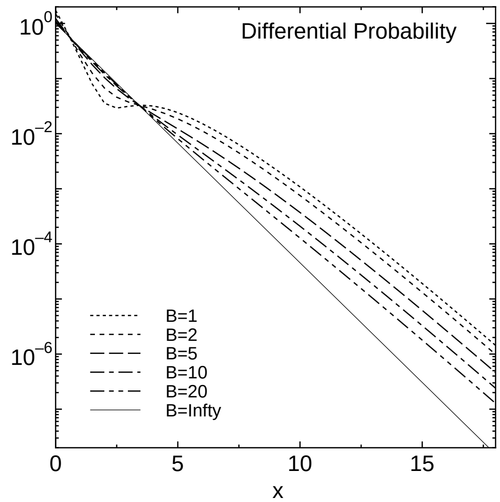

Fig. 1. Central distribution produced by the moderate scattering, divided at χ_B. x denotes ϑ^2.

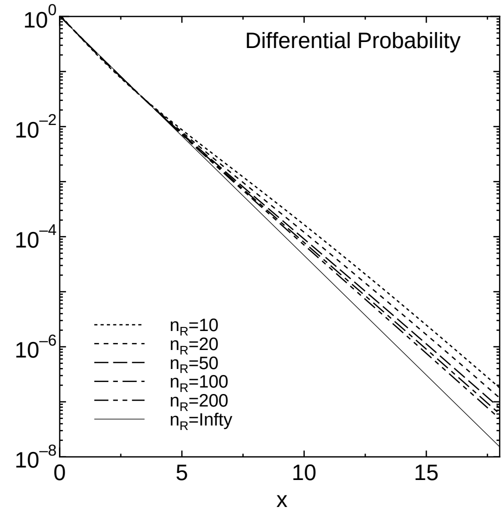

Fig. 2. Central distribution produced by the moderate scattering, divided at χ_C. x denotes ϑ^2.

3. Smooth continuation between the Rutherford sampling and the splitting cross-section sampling

The splitting cross-section sampling is formulated by Poisson probability distribution as

$$f(\vartheta)d\vec{\vartheta} = e^{-p}d\vec{\vartheta}\sum_{k=0}^{\infty}\frac{1}{k!}p^k N * \sigma_{\mathrm{L}}^{(k)}, \tag{5}$$

where p denotes the mean number of large-angle scattering σ_{L} within the depth and $N * \sigma_{\mathrm{L}}^{(k)}$ denotes the folding integral of the central distribution N produced by the moderate scattering σ_{M} and the k-times large-angle scattering $\sigma_{\mathrm{L}}^{(k)}$.

It is well known that the Molière formula breaks at very thin thicknesses, where N becomes far different from gaussian distribution or splitting angle becomes smaller than the screening angle. In these thicknesses we apply the Rutherford sampling formulated as

$$f(\vartheta)d\vec{\vartheta} = e^{-p}d\vec{\vartheta}\sum_{k=0}^{\infty}\frac{1}{k!}p^k \sigma_{\mathrm{R}}^{(k)}, \tag{6}$$

where $\sigma_{\mathrm{R}}^{(k)}$ denotes the k-times single scattering and p denotes the mean number of single scattering σ_{R} within the depth. We show in Fig. 3 smooth continuation between those, where we took χ_{C} of $p = 1$ as the splitting angle.

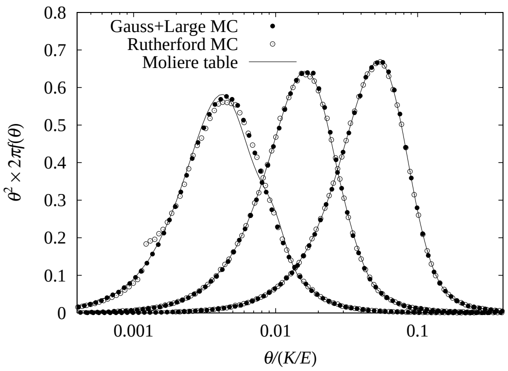

Fig. 3. Continuation of Molière angular distributions derived by the Rutherford sampling and our χ_C-cut sampling. $t = e^{2k+1}\Omega e^{-\Omega}$ with $k = 1, 2, 3$, from left to right.

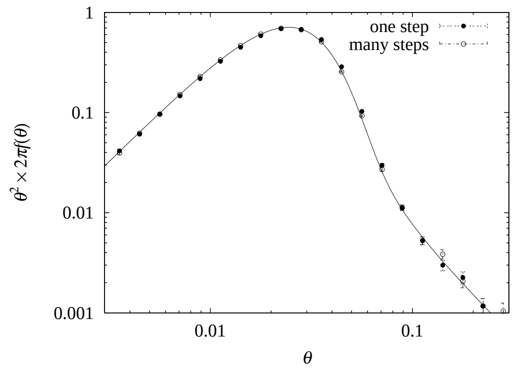

Fig. 4. Molière angular distributions for 100 GeV muon with E-loss of 90%, derived by our χ_C-cut sampling and the traditional sampling.

4. Multiple-scattering sampling under the ionization process

The most characteristic aspect of our simulation code will be the high-accuracy and the high-efficiency of the method, based on the Molière theory with ionization [5]. We can get the Molière angular distribution with ionization by one sampling sequence. We had to separate the penetrating passage into many short stepsizes in the traditional method so as energies of particle not to be changed much within the individual step. The Molière angular distribution with ionization derived through our sequence agrees well with that through the traditional method, even with E-loss of 90% as indicated in Fig. 4.

5. Conclusions and discussions

A new simulation code for the multiple Coulomb scattering process is developed, based on the Molière theory with ionization. We use the splitting cross-section method dividing at χ_C, where we have confirmed the moderate scattering produces the gaussian distribution and the additive large-angle scattering above χ_C well reproduces the Molière angular distribution in enough high accuracy. Smooth continuation has been confirmed between the Rutherford sampling at very thin thicknesses and the splitting cross-section sampling at ordinary thicknesses. Accuracies and efficiencies of simulation for the Molière process has been extremely improved especially under the ionization process.

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