Splitting Model of the Single Scattering to Reconstruct the Molière Process of Multiple Coulomb Scattering

Takao Nakatsuka
Okayama Shoka University, Tsushima-Kyomachi, Okayama 700-8601, Japan

Abstract

We have examined the properties of Molière multiple-scattering process by dividing the single scattering into the high-frequency moderate scattering and the low-frequency large-angle scattering. We took the splitting angle of the cross-section as an arbitrary constant and investigated the resultant configuration of angular distribution by using Kamata-Nishimura formulation of the Molière theory.

1. Introduction

Owing to the Kamata-Nishimura formulation or the differential formulation of Molière theory [1,2], it has become far easy for us to investigate properties of multiple Coulomb scattering process in general. We discuss this time the results of Molière process derived through single-scattering splitting method and investigate dynamic properties of Molière process under both the fixed-energy and the ionization processes.

2. Arbitrary splitting of the single-scattering at $\chi_{\rm B}'$ and an extended formulation of Molière process

We start with the single scattering cross-section under the extreme relativistic condition [3]:

$$\sigma(\theta)2\pi\theta d\theta dt = \frac{1}{\pi\Omega}\frac{K^2}{E^2}\theta^{-4}2\pi\theta d\theta dt \quad \text{with} \quad \theta > \sqrt{e}\chi_{\rm a}. \tag{1}$$

We introduce a splitting angle $\chi_{\rm B}'$ to divide the cross-section σ into the moderate scattering $\sigma_{\rm M}$ and the large-angle scattering $\sigma_{\rm L}$ as indicated in Fig. 1:

$$\sigma(\theta) = \sigma_{\rm M}(\theta) + \sigma_{\rm L}(\theta), \quad \text{with} \tag{2}$$

$$\sqrt{e}\chi_{\rm a} = (K/E)e^{-\Omega/2+1-C}, \quad \text{and} \quad \chi_{\rm B}' \equiv e^{B'/2}\sqrt{e}\chi_{\rm a}. \tag{3}$$

Then the diffusion equation

$$\frac{d}{dt}f(\vec{\theta},t) = \iint \{f(\vec{\theta}-\vec{\theta}',t) - f(\vec{\theta},t)\}\sigma(\vec{\theta}')d\vec{\theta}' \tag{4}$$

pp. 1491–1494

is described in the frequency space as

$$\frac{d\tilde{f}}{dt} = 2\pi\tilde{f}\int_0^\infty [J_0(\zeta\theta) - 1]\{\sigma_{\rm M}(\theta) + \sigma_{\rm L}(\theta)\}\theta d\theta. \tag{5}$$

The integration is evaluated in the first order by using Eq. (14) of Bethe [4], as

$$\int_0^\infty [J_0(\zeta\theta) - 1]\sigma_{\rm M}(\theta)2\pi\theta d\theta \simeq -\frac{B'}{\Omega}\frac{K^2\zeta^2}{4E^2}, \tag{6}$$

$$\int_0^\infty [J_0(\zeta\theta) - 1]\sigma_{\rm L}(\theta)2\pi\theta d\theta \simeq \frac{1}{\Omega}\frac{K^2\zeta^2}{4E^2}\ln(\frac{K^2\zeta^2}{4E^2}e^{B'-\Omega}). \tag{7}$$

So we get an extended Molière equation corresponding to the arbitrary splitting of the single-scattering:

$$\frac{d\tilde{f}}{dt} = -\frac{B'}{\Omega}\frac{K^2\zeta^2}{4E^2}\tilde{f}\{1 - \frac{1}{B'}\ln(\frac{K^2\zeta^2}{4E^2}e^{B'-\Omega})\}. \tag{8}$$

Under the ionization process of a constant rate, there hold

$$\int_0^t \frac{K^2}{E^2}dt = \frac{K^2 t}{E_0 E} \quad \text{and} \quad \int_0^t \frac{K^2}{E^2}\ln\frac{K^2}{E^2}dt = \frac{K^2 t}{E_0 E}\ln\frac{K^2}{\nu E_0 E} \tag{9}$$

$$\text{with} \quad \nu = e^2(E/E_0)^{(E_0+E)/(E_0-E)}. \tag{10}$$

Thus we have

$$\tilde{f} = \frac{1}{2\pi}\exp\{-\frac{\theta_{\rm M}'^2\zeta^2}{4}(1 - \frac{1}{B'}[\ln\frac{\theta_{\rm M}'^2\zeta^2}{4} - \ln\tau])\} \quad \text{with} \tag{11}$$

$$\theta_{\rm M}'^2 \equiv \frac{B'}{\Omega}\theta_{\rm G}^2 = \frac{B'}{\Omega}\frac{K^2 t}{E_0 E} \quad \text{and} \quad \tau \equiv \frac{\nu}{E/E_0}\frac{\theta_{\rm M}'^2}{\chi_{\rm B}'^2}e^{2-2C}. \tag{12}$$

Using $\vartheta \equiv \theta/\theta'_{\rm M}$, we have the angular distribution in a double series with B'^{-1} and $\ln\tau$, similar as Eq. (5) of our preceding paper [5]:

$$\begin{aligned} 2\pi f(\vartheta) \;=\; & f^{(0)}(\vartheta) + \frac{1}{B'}\{f^{(1)}(\vartheta) + f_1^{(1)}(\vartheta)\ln\tau\} \\ & + \frac{1}{B'^2}\{f^{(2)}(\vartheta) + f_1^{(2)}(\vartheta)\ln\tau + f_2^{(2)}(\vartheta)(\ln\tau)^2\} + \cdots. \end{aligned} \tag{13}$$

3. An adequate splitting to interpret Molière expansion

As B' was remained as an arbitrary constant, we can take B' so as $\ln\tau$ vanishes, or

$$\chi_{\rm B}'^2 = \frac{\nu}{E/E_0}\theta_{\rm M}'^2 e^{2-2C}. \tag{14}$$

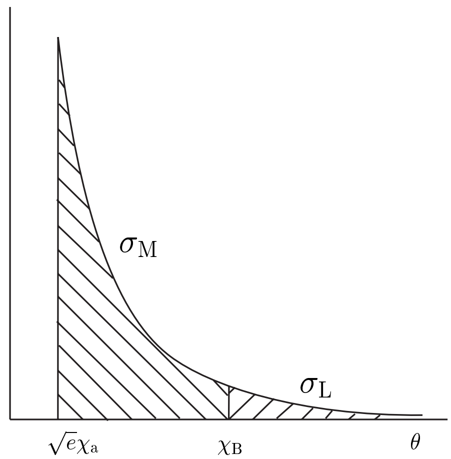

Fig. 1. Separation of the single scattering σ at χ_{B} to the moderate scattering σ_{M} and the large-angle scattering σ_{L}.

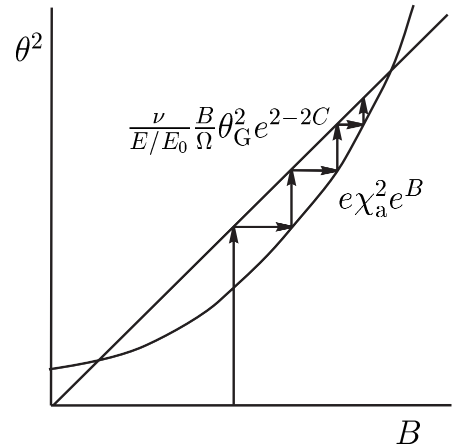

Fig. 2. The expansion parameter B can be determined by a successive method.

Substituting Eqs. (3) and (12), we find Eq. (14) gives

$$B - \ln B = \Omega - \ln \Omega + \ln(\nu t) \tag{15}$$

to determine the exact expansion parameter B, as well as the adequate scale angle θ_{M} and the separation angle χ_{B} of Molière case:

$$\theta_{\mathrm{M}} = \theta_{\mathrm{G}}\sqrt{B/\Omega} \quad \text{and} \quad \chi_{\mathrm{B}} \equiv e^{B/2}\sqrt{e}\chi_{\mathrm{a}}. \tag{16}$$

Then Eq. (13) becomes the simple single series of Molière:

$$2\pi f(\vartheta) = f^{(0)}(\vartheta) + B^{-1}f^{(1)}(\vartheta) + B^{-2}f^{(2)}(\vartheta) + \cdots. \tag{17}$$

4. Successive derivation of B and an interpretation of multiple scattering process

Molière B can be also derived by a successive method by Eq. (14), as indicated in Fig. 2. Qualitative feature of the Molière B is studied from a first approximation of B derived by substituting θ_{G} instead of θ'_{M} on the right-hand side of Eq. (14).

Under the fixed-energy conditions, Eq. (14) becomes

$$\chi_{\mathrm{B}}'^2 = \theta_{\mathrm{M}}'^2 e^{2-2C} \tag{18}$$

and θ_{G} is expressed by $K\sqrt{t}/E$. So the first approximation of χ_{B} increases monotonously as $t^{1/2}$ with the traversed thickness, on the other hand $\sqrt{e}\chi_{\mathrm{a}}$ stays constant. B' is defined in (3) by a half of logarithm of the ratio $\chi'_{\mathrm{B}}/\sqrt{e}\chi_{\mathrm{a}}$.

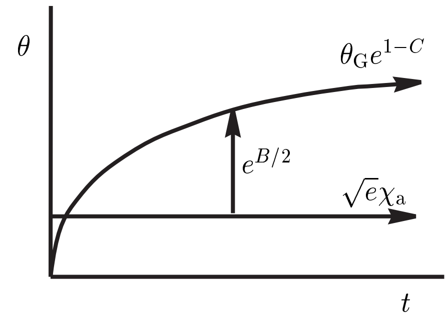

Fig. 3. B is evaluated by putting $\theta_{\mathrm{M}} = \theta_{\mathrm{G}}$ as a first approximation under the fixed-energy process.

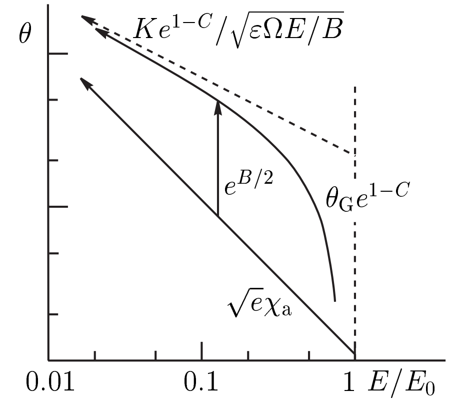

Fig. 4. B is evaluated by neglecting $\nu/(E/E_0)$ and putting $\theta_{\mathrm{M}} = \theta_{\mathrm{G}}$ as a first approximation under the ionization process.

So we can understand B increases monotonously with the traversed thickness, as indicated in Fig. 3. Under the ionization process, both ν and E/E_0 in Eq. (14) decrease monotonously with the traversed thickness. So a rough estimation of B could be made by neglecting the factor $\nu/(E/E_0)$ and substituting $\theta_{\mathrm{G}}^2 = K^2 t/(E_0 E)$ into $\theta_{\mathrm{M}}'^2$ on the right-hand side. $\sqrt{e}\chi_{\mathrm{a}}$ increases proportionally to E^{-1} with dissipation of energy in this condition, on the other hand θ_{G} increases more rapidly at first stage but later it increases more slowly as proportionally to $E^{-1/2}$ than $\sqrt{e}\chi_{\mathrm{a}}$, as indicated in Fig. 4. So B increases at first stage of traverse, nevertheless it begins to decrease in the latter stage under the ionization process.

5. Conclusions and discussions

We found the Molière expansion corresponds to the splitting angle at $e^{B/2}$ times the screening angle, when the moderate scattering gives the central gaussian distribution and the large-angle scattering only gives the tail-angle distribution and does not affect the shape of the central gaussian distribution. Mechanism of depth-variation of Molière angular distribution both under the fixed-energy and the ionization processes is also interpreted by the method. Formulations developed here will be also valuable for investigations of Monte Carlo code using the splitting cross-section method [6,7,8].

References

[1] K. Kamata and J. Nishimura, Prog. Theor. Phys. Suppl. **6**, 93(1958).

[2] J. Nishimura, in *Handbuch der Physik, Band 46*, edited by S. Flügge (Springer, Berlin, 1967), Teil **2**, p. 1.

[3] T. Nakatsuka, *Proc. of the 26th ICRC*, Salt Lake City, **1**, 455(1999).

[4] H.A. Bethe, Phys. Rev. **89**, 1256(1953).

[5] T. Nakatsuka, *Proc. of the 26th ICRC*, Salt Lake City, **1**, 522(1999).

[6] T. Nakatsuka and K. Okei, in this conference.

[7] K. Okei and T. Nakatsuka, in this conference.

[8] J.M. Fernandez-Varea, R. Mayol, J. Baro, and F. Salvat, Nucl. Instrum. Methods Phys. Res. **B73**, 447(1993).