The three-dimensional propagation of high energy muon through water

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Abstract

We examin the three-dimensional propagation of the high energy muon with higher than 500 GeV through water which are recognized as [Partially Contained Event].

1. Introduction

Now, the construction of 1km^3 detector for high or extremely high energy neutrino astrophysics are under discussion[1][2]. However, it should be noticed that muon events with higher energy than 500 GeV could not be [Fully Contained Event] even for 1km^3 detector, while even electron event with 10^{20} eV could be [Fully Contained Event]. Consequently, we should get information on the muon events as much as possible, expecting decrease in ambiguities. In the present paper, we examin three-dimensional propagation of high energy muon as exactly as possible, emphasizing the importance of flactuation effect.

2. Procedure for Calculation

In present paper, we study three-dimensional propagation of high energy muon in water. High energy muons suffer from fluctuation effect so much due to stochastic nature of physical processes concerned, direct pair production , bremsstrahlung and nuclear interaction. In our calculation, we fully take into account these fundamental processes and simulate in a rigorous way. We consider multiple scattering to clarify three-dimensional behavior of high energy muon. Let us describe our simulation procedure. The total mean free path of the muon from direct electron pair production, bremsstrahlung and nuclear interaction, is given as

$$\frac{1}{\lambda} = \frac{1}{\lambda_p} + \frac{1}{\lambda_b} + \frac{1}{\lambda_n} \tag{1}$$

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where inclusion of the exitation and ionization into λ is carried out with the method by Tamura [3] and λ_p, λ_b and λ_n denotes the mean free path for direct electron pair production, bremsstrahlung and nuclear interaction, respectively. The total mean free path of the muon obeys the following distribution

$$P(t,\lambda)dt = \frac{1}{\lambda}\exp\left(-\frac{t}{\lambda}\right)dt \tag{2}$$

Now, let us describe our procedure for range fluctuation concretely.

2.1. The procedure 1

We decide the interaction point of the muon concerned, utilizing random uniform number between 0 and 1 in the following:

$$t = -\ln\xi \tag{3}$$

2.2. The procedure2

We decide the kind of the interaction concerned explicitly. Here, we define the following two values in (4) and (5)

$$\xi_p = \frac{1/\lambda_p}{1/\lambda_p + 1/\lambda_b + 1/\lambda_n} \tag{4}$$

$$\xi_b = \frac{1/\lambda_p + 1/\lambda_b}{1/\lambda_p + 1/\lambda_b + 1/\lambda_n} \tag{5}$$

2.3. The procedure 3

We obtain, ξ_k, a new random number between 0 and 1. Now, if $\xi_k \leq \xi_p$, then we judge that the Iinteraction concerned is attributed to the direct pair production. If $\xi_p < \xi_k \leq \xi_b$, then we judge that the interaction concerned is attributed to the bremsstrahlung. If $\xi_k > \xi_b$, then we judge that the interaction is attributed to the nuclear interaction.

2.4. The procedure

Once we decide the origin of the interaction concerned, we decide, the dissipated energy by the muon due to the interaction concerned. For the purpose, we choose a new uniform random number between 0 and 1 in the following.

$$\xi = \frac{\int_{E_{min}}^{E_d} \psi_{p,b,n}(E_\mu, E_d)dE_d}{\int_{E_{min}}^{E_{max}} \psi_{p,b,n}(E_\mu, E_d)dE_d} \tag{6}$$

,where $\psi_{p,b,n}(E_\mu, E_d)dE_d$ denote the differential cross section for the interaction concerned and E_d is the dissipated energy by the muon due to the interaction concerned. Through the procedure 1 to the procedure 4, we decide the interaction point and dissipated energy of the muon due to the interaction concerned . By the repetition of the procedure 1 to 3, the muon goes on, dissipating its energy due to the interaction concerned and finally dying, decaying electron.

3. Results

In Figure 1, the longtudinal behavour of a muon with 10^{15} eV is shown, taking into account of the stochasetic character in direct pair production, bremsstrahlung and nulclear interaction exactly. The ordinte axis denots the ratio of the dissipated energy to primary muon energy . The bresstrahlung and nuclear interaction are of much stochastic character compared with direct pair production, as shown in the figure. In Figure 2, the range distributions of high and extremely high energy muons are given for 1 Tev, 100 Tev, 10^4 TeV, 10^6 TeV, and 10^8 TeV. It is easily understood that fluctuation in range increases, as primary energy increases. The increase of the fluctuation in the increase of primary energy in longitudinal development predicts us that the fluctuation in the lateral direction become strong as primary energy increases. The fact invites difficulty in the interpretation to [Partially Contained Event]

If we incorporate multiple scattering theory [4] into the longitudinal behavior of muons, we obtain three-dimensional propagation of muon at high energies. Namely, we obtain the probability function for lateral spread as well as the probability function for angular spread at definite depths. As we mentioned to the Introduction, almost high energy muon higher than 500 GeV become [Partially Contained Events]

4. References

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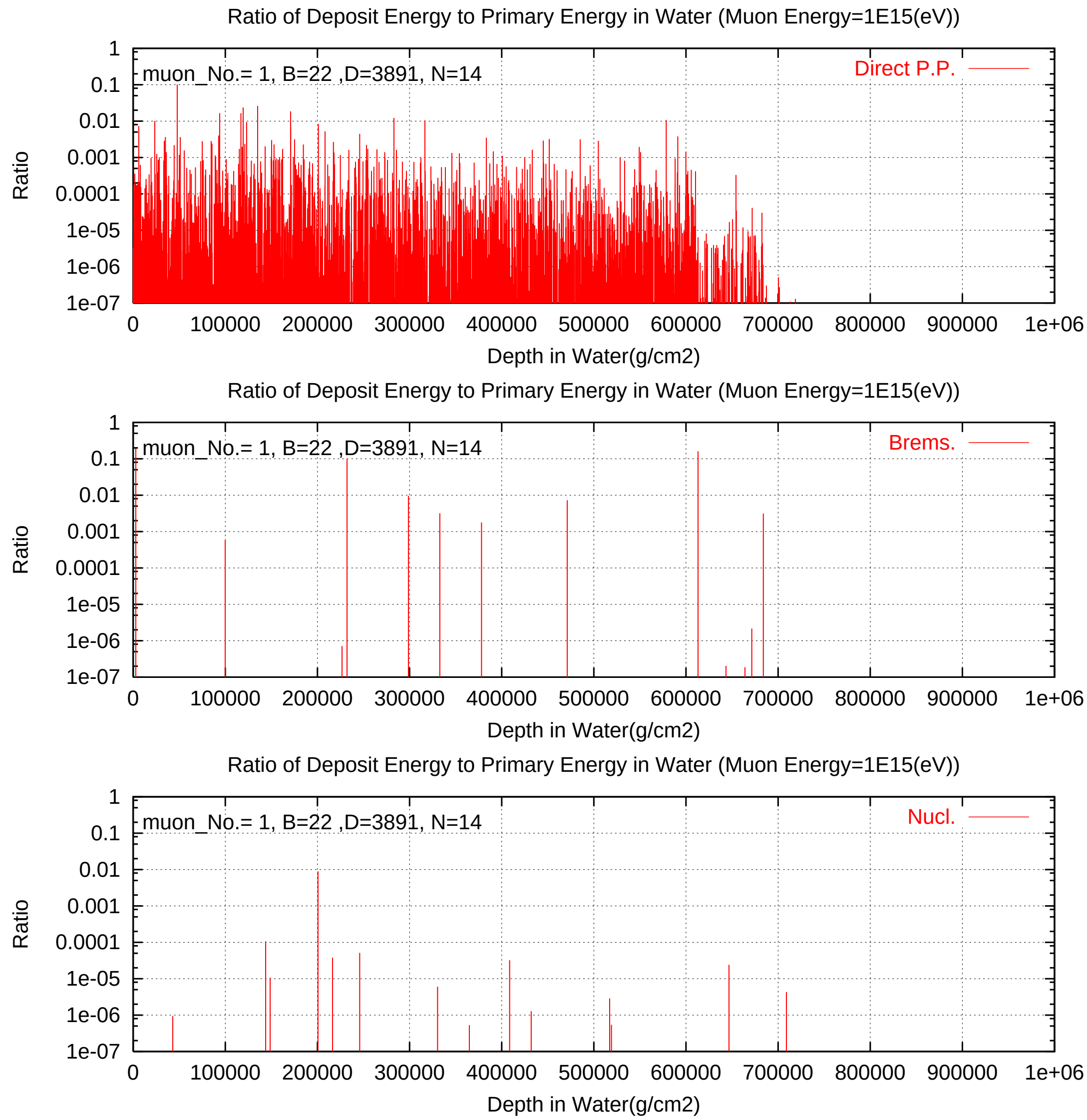
Ratio of Deposit Energy to Primary Energy in Water (Muon Energy=1E15(eV))
muon_No.= 1, B=22 ,D=3891, N=14
Direct P.P.
Ratio
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
100000
200000
300000
400000
500000
600000
700000
800000
900000
1e+06
Depth in Water(g/cm2)
Ratio of Deposit Energy to Primary Energy in Water (Muon Energy=1E15(eV))
muon_No.= 1, B=22 ,D=3891, N=14
Brems.
Ratio
Depth in Water(g/cm2)
Ratio of Deposit Energy to Primary Energy in Water (Muon Energy=1E15(eV))
muon_No.= 1, B=22 ,D=3891, N=14
Nucl.
Ratio
Depth in Water(g/cm2)

Fig. 1.

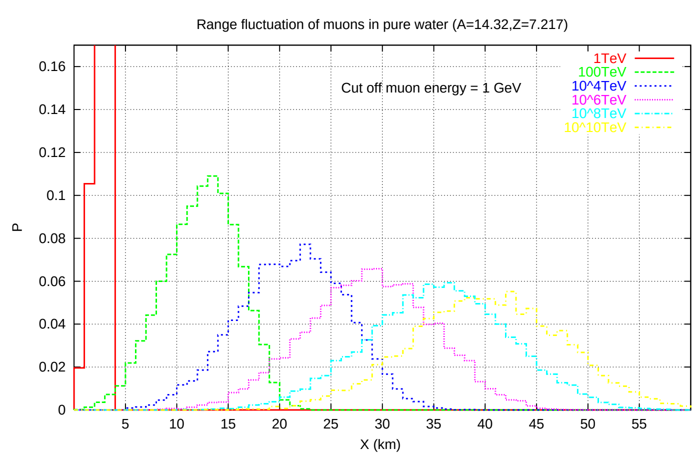
Range fluctuation of muons in pure water (A=14.32,Z=7.217)
Cut off muon energy = 1 GeV
1TeV
100TeV
10^4TeV
10^6TeV
10^8TeV
10^10TeV
P
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
5
10
15
20
25
30
35
40
45
50
55
X (km)

Fig. 2.