The Cross-Section of Muon Photo-Nuclear Interaction

A.V.Butkevich and S.P.Mikheyev
Institute for Nuclear Research of Russian Academy of Science, 60th October Anniversary prospect, 7a, Moscow 117312, Russia

Abstract

Formula for the cross-section differential in energy transferred by muon in inelastic interaction on nucleus was derived. Is is approximation of the corresponding cross-section that was re-evaluated taking into account HERA results at low and moderate value of Q^2. The results of calculations of the total cross-section and muon energy loss due to photo-nuclear interaction in standard rock are shown.

1. Introduction

The inelastic interaction of high-energy muon is theoretically much less understood than purely electromagnetic processes. The reason is that the bulk of this process is characterized by low squared four-momentum transfer Q^2. The smallness of the Q^2 does not allow to use of the perturbative QCD (pQCD) for calculation of nuclear structure function (SF) and phenomenological modeles such as the Regge and General Vector Dominance Model (GVDM) have to be used. The parametrization of the nucleon SF obtained in this models, depends on free parameters which can be determined from a fit of experimental data and can be applied in the range of $Q^2 \leq 1 - 3$ GeV^2. Recently, precise data on SF in wide ranges of Q^2 have been obtained by H1 and ZEUS collaborations.

The muon-nucleus inelastic cross-section $d^2\sigma_{\mu A}/d\nu dQ^2$ (ν is muon's energy loss) has been calculated [1], based on the modern nucleon SFs that describe well these data. The nuclear effects (anti-shadowing and EMC-effect) were taking into account also. But the full expression of $d^2\sigma_{\mu A}/d\nu dQ^2$ has rather complicate form. In this work we give a simple formula for calculation of $d\sigma_{\mu A}/d\nu$.

2. Low-Q^2 approximation of the structure functions

The CKMT [2] model proposes the following parameterization of the proton structure function F_2^p

$$F_2^p(x, Q^2) = F_S^p(x, Q^2) + F_{NS}^p(x, Q^2). \tag{1}$$

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The singlet term is given by

$$F_S^p(x,Q^2) = A_S x^{-\Delta(Q^2)} (1-x)^{n(Q^2)+4} \left(\frac{Q^2}{Q^2+a}\right)^{1+\Delta(Q^2)}, \tag{2}$$

where $\Delta(Q^2) = \Delta_0 [1+2Q^2/(Q^2+d)]$, $n(Q^2) = 1.5 [1+Q^2/(Q^2+c)]$, $x = Q^2/M\nu$ and M is nucleon's mass. The parameterization of non-singlet term is

$$F_{NS}^p(x,Q^2) = B x^{(1-\alpha_R)} (1-x)^{n(Q^2)} \left(\frac{Q^2}{Q^2+b}\right)^{\alpha_R}. \tag{3}$$

The valence quark distribution can be separated into the contributions of the u and d valence quarks by replacing

$$F_{NS}^p(x,Q^2) = xU_V(x,Q^2) + xD_V(x,Q^2), \tag{4}$$

$$xU_V(x,Q^2) = B_u x^{(1-\alpha_R)} (1-x)^{n(Q^2)} \left(\frac{Q^2}{Q^2+b}\right)^{\alpha_R}, \tag{5}$$

$$xD_V(x,Q^2) = B_d x^{(1-\alpha_R)} (1-x)^{n(Q^2)+1} \left(\frac{Q^2}{Q^2+b}\right)^{\alpha_R}. \tag{6}$$

Normalization conditions for valence quarks in proton fix the values of parameters B_u and B_d at $Q^2 = Q_0^2$.

The general expression for neutron SF F_2^n is given by Eq.(1), where

$$F_S^n(x,Q^2) = A_S x^{-\Delta(Q^2)} (1-x)^{n(Q^2)+\tau} \left(\frac{Q^2}{Q^2+a}\right)^{1+\Delta(Q^2)}, \tag{7}$$

$$F_{NS}^n(x,Q^2) = \frac{1}{4} xU_V(x,Q^2) + 4xD_V(x,Q^2), \tag{8}$$

and $xU_V(x,Q^2)$ and $xD_V(x,Q^2)$ are given by Eq.(5) and Eq.(6).

To determine the parameters we have made a joint fit of the $\sigma_{tot}^{\gamma p}$ data and NMC, E665, SLAC, ZEUS, and H1 data on the proton and deuteron SFs F_2 in the region $0.11 \le Q^2 \le 5.5$ GeV^2 and $10^{-6} \le x \le 0.98$ [3]. A global fit results in the following values of parameters (all dimensional parameters are in GeV^2): a=0.2513, b=0.6186, c=3.0292, d=1.4817, Δ_0=0.0988, α_R=0.4056, $\tau = 1.8152$ and A_S=0.12. The values of the parameters $B_u = 1.2437$ and $B_d = 0.1853$, were determined from normalization conditions for valence quarks (at Q_0^2=2 GeV^2).

The SFs measured for different nuclei A (charge Z) are found to differ from the SF measured on nucleon. In region of low Q^2 (low-x) the nuclear SF can be written as follow

$$F_2^A(x,Q^2) = G(\eta) \left[Z F_2^p(x,Q^2) + (A-Z) F_2^n(x,Q^2) \right], \tag{9}$$

where the function $G(\eta)$ is

$$G(\eta) = 3\left[0.5\eta^2 - 1 + e^{-\eta}(1+\eta)\right]/\eta^3, \tag{10}$$

$\eta = 0.00282A^{1/3}\sigma_{\gamma N}(\nu)$ and the averaged photon-nucleon cross-section one can write as

$$\sigma_{\gamma N}(\nu) = (\sigma_{\gamma p} + \sigma_{\gamma n})/2 = 112.2\left(0.609\nu^{0.0988} + 1.037\nu^{-0.5944}\right). \tag{11}$$

3. Approximate formula

Taking into account that the main contribution to the total muon inelastic cross-section is due to photo-production (low Q^2 process) we can approximate the spectra of muon energy loss (N_{Av} is Avogadro's number) in single interaction as,

$$\frac{N_{Av}}{A}\nu\frac{d\sigma_{\mu A}}{d\nu} = \frac{N_{Av}}{A}\nu\int\limits_{Q^2_{min}}^{Q^2_{max}}\frac{d\sigma^2_{\mu A}}{d\nu dQ^2}dQ^2, \tag{12}$$

where

$$\frac{d\sigma^2_{\mu A}}{d\nu dQ^2} = 2\pi\alpha^2\frac{F_2^A(x,Q^2)}{Q^2}\left\{\frac{\nu(1+Q^2/\nu^2)}{E^2Q^4(1+R)}(Q^2-2m^2) + \frac{2(E-\nu)}{Q^2E\nu} - \frac{1}{\nu E^2}\right\}, \tag{13}$$

m is muon's mass and R=0.25. The allowed kinematical region for the variables ν and Q^2 is determined by the following equations:

$$\nu_{min} = (S_x - M^2)/2M, \qquad \nu_{max} = E - m, \qquad S_x = (M+m_\pi)^2, \tag{14}$$

$$Q^2_{min} = m^2\nu/(E-\nu), \quad Q^2_{max} = \min\{2M\nu - M^2 - S_x,\ 5.5\ GeV^2\}, \tag{15}$$

where m_π is pion's mass. This approximation may be used up to muon energy $E = 10^6$ GeV and $Q^2 = 5.5$ GeV^2. In the region $E = 10 \div 10^3$ GeV its precision is ≈5% and ≈2% at $E \geq 10^4$ GeV. The energy dependence of the total cross section,

$$\sigma_{\mu A}(E) = \int\limits_{\nu_{min}}^{\nu_{max}}\frac{d\sigma_{\mu A}}{d\nu}d\nu, \tag{16}$$

and muon energy loss,

$$b_n(E) = \frac{N_{Av}}{A}\int\limits_{\nu_{min}}^{\nu_{max}}\nu\frac{d\sigma_{\mu A}}{d\nu}d\nu \tag{17}$$

in standard rock are shown in Fig.1. The results from Ref.[4] are given in these figures for comparison also.

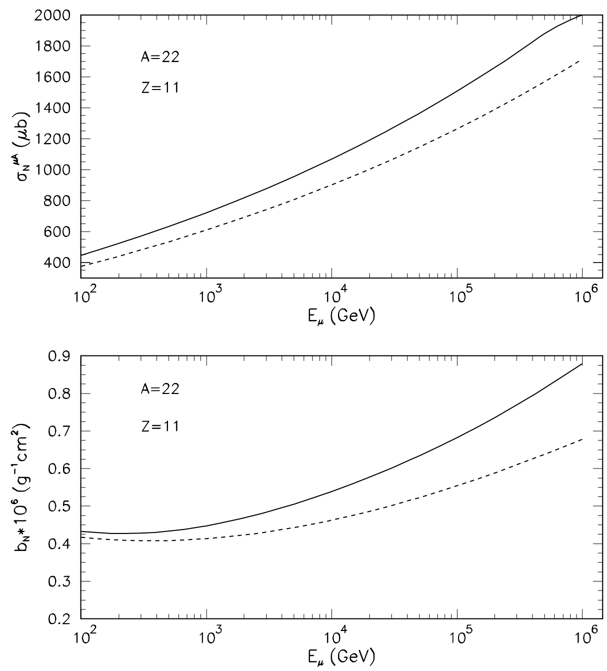

Fig. 1. The total cross-section $\sigma_{\mu A}$ and muon energy loss b_n for muon photo-nuclear interaction in standard rock as a function of muon energy E, as compared to those from Ref.[4] (dashed lines).

4. Conclusions

The cross-section of inelastic muon scattering obtained in the present work is larger by factor 1.2 and the muon energy losses $b_n(E)$ is larger too by ≈8% at E =10^3 GeV and by ≈30% at E =10^6 GeV. These differences are mainly due to contributions of small x and small Q^2 where the modern SFs are large than SFs used by Bezrukov and Bugaev [4].

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6. References

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