On the Binning of Atmospheric Neutrino Fluxes Near the Horizon in Monte–Carlo Calculations

G. Barr,[1] T. K. Gaisser,[2] S. Robbins,[1] & T. Stanev[2]
(1) Department of Physics, University of Oxford, UK
(2) Bartol Research Institute, University of Delaware, Newark, DE, USA

Abstract

It is desirable to use a flat detector when computing atmospheric neutrinos with a three dimensional Monte-Carlo technique. We investigate the statistical implications of doing this and evaluate the performance of two modifications to the approach.

1. Introduction

To keep in step with the continued improvement in the quality of underground neutrino data [5,7,8], atmospheric neutrino calculations are being developed with increasing sophistication. While the main sources of error in predicting the rate of neutrino interactions underground remain the hadroproduction uncertainties, the neutrino cross sections and the primary fluxes, the calculations have been developed to an extent where the one-dimensional approximations of early calculations (e.g. [1]) have been removed [3,4,9].

In moving to three dimensions (3D), a number of conveniences must be abandoned. In particular, in one dimensional simulations (1D), all neutrinos which are generated automatically reach the detector by definition. The 1D flux is automatically expressed in units containing the correct unit area perpendicular to the incoming neutrino direction. In 3D, the particles are generated according to the distributions in which they occur naturally. To save computer time, the detectors are artificially enlarged. Tserkovnyak et. al. [9] observe that it is undesirable to extend the detector vertically as this creates distortions in the fluxes. The procedure adopted in our calculation [2] and in others is to use a detector area which is extended across the surface for O(1000km) and is 'locally flat', i.e. it follows the curvature of the earth, but not the local terrain. In converting from the number of particles incident on a detector defined in this way to a flux given in units per unit area, a number of issues arise which are discussed in the following sections of this paper.

pp. 1423–1426 ©2003 by Universal Academy Press, Inc.

2. Weighting

The use of a flat detector requires that the events are weighted by a factor $1/\cos\theta$, where θ is the zenith angle of the neutrino, to obtain fluxes in per-unit-area units. Near the horizon, the number of neutrinos in the simulation becomes small and the weights are large. A single neutrino may dominate the statistics. These uneven features of the fluxes are difficult for experiments to use in their data analyses. Additionally, high weight neutrinos often contribute twice, (once as a downward neutrino and then as an upward neutrino) as they graze the slightly curved detector which hugs the surface of the earth. This correlation of the high-weight events is also undesirable.

We investigate this problem and two alternative schemes which overcome it using a simple Monte-Carlo. The Monte-Carlo generates an isotropic flux hitting a flat detector by generating events with a probability proportional to $\cos\theta$, and thus requiring weights of $1/\cos\theta$ to recover the isotropic flux. This simulation is run 100 times with separate random seeds, each run to a total of 10,000 neutrinos. The neutrinos are binned in $\cos\theta$ with 20 bins each of size 0.1 between $\cos\theta$ of -1.0 and 1.0. The expected number of neutrinos after weighting is therefore 1000 in each bin. The bin closest to the vertical ($\cos\theta$ between 0.9 and 1.0) has an expected contents of 950 neutrinos (before weighting) whereas the bin closest to the horizontal has an expected contents of 50 neutrinos. These runs are a factor of about 100 shorter than in a real 3D Monte-Carlo run in order to emphasize the statistical problems.

Figure 1(a) shows the weighted contents of the horizontal ($\cos\theta$ between 0.0 and 0.1) bin for each of the 100 runs. The points have been sorted in increasing bin contents to ease the statistical measurements on the graph. The error bars are sample standard deviation errors, i.e. estimated from the events in the bin in each run separately. Two undesirable features of the distributions are apparent (a) there is a high end tail in the distribution which occurs when a statistical fluctuation produces a particularly grazing track which contributes a large weight and (b) the sample error estimates are larger for the Monte-Carlo runs where the flux fluctuates higher and lower for the runs where the flux was lower.

3. Modified individual weights

The first of the alternative weighting schemes to be investigated is one in which the individual $1/\cos\theta$ weights are maintained, where events below $|\cos\theta| = 0.01$ are completely ignored and where events in the interval $|\cos\theta|$ from 0.01 to 0.1 have their weights increased by 10% to compensate. This has the effect of cutting the tiny fraction of events with the highest weights, and where the statistical fluctuations are highest and using the adjacent region in $\cos\theta$ to estimate the size of the contribution which was removed. In these runs to 10,000 events, the

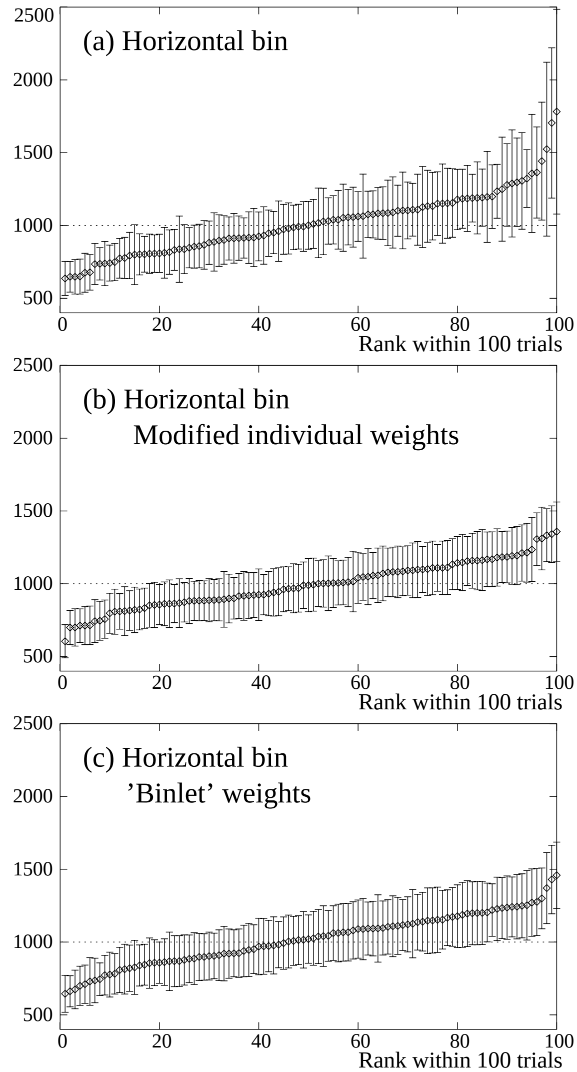

Fig. 1. Sorted value and error estimates from the simple Monte Carlo.

expected number of events to be dropped by this algorithm is 0.5. Figure 1(b) shows the sorted distributions using this algorithm for the same 100 runs. The error bars are again estimated from the sample. The tail at the high end is considerably reduced and the error bars are more constant along the whole range of values.

There is no bias in this scheme provided the flux is flat across the interval $|\cos\theta|$ from 0.0 to 0.1. A simple analytical estimate of the bias when there is a constant slope in the flux across the bin is $1+\rho\gamma$ where ρ is the fractional difference in the flux between the edges of the bin and the centre. γ is the fraction of the range between 0 and 0.1 which is removed (0.1 in the above example). The 3D effects due to geometrical considerations at low energy described by Lipari [6] are responsible for the largest variation in fluxes near the horizon which can be as large as $\rho = 0.15$ for bins of width $\Delta \cos\theta = 0.1$. The largest feasible bias from this method is therefore a change of about 1.5%. The value of γ was chosen by finding that the optimum from statistical considerations was rather flat between $\gamma = 0.1$ and $\gamma = 0.5$. $\gamma = 0.1$ was therefore chosen to reduce the bias.

4. Binned weights

The second weighting scheme is based on binning the events and then applying the same weight to all the events in a bin. The weight is taken as $1/\cos\theta$ of the bin-centre. If the flux is flat across the bin, there is no bias in this estimator, despite the varying event population across the bin. However, when a slope defined by ρ is introduced, the bias is found to be $1 + \rho/3$. For bins with $\Delta \cos\theta = 0.1$, where ρ can be as large as 0.15, the bias is around 5% which is a bit large. We therefore use the technique with 'binlets' which are a quarter this size. The technique is to assign each neutrino to one of 80 binlets in $\cos\theta$ and to use a weight corresponding to the middle of the binlet. ρ is now about 0.04 and the bias is about 1.3% which is again acceptable. There are 40 unique values of weights in use from 1.013 near the vertical to 80.000 near the horizontal. Figure 1(c) shows the sorted distributions for the 100 Monte-Carlo runs. The method works about as well as the modified individual weight method.

5. Conclusions

We have shown that care is needed in binning in Monte-Carlos which use a flat detector for events near the horizon to avoid weighting effects. It is easier for experiments to make use of the fluxes if they do not have fluctuations due to individual events grazing the detector close to $\cos\theta = 0$. We have developed two techniques which address this issue and show that both behave satisfactorily. Both techniques have been implemented in our full 3D simulation programme and work as expected.

1. Agrawal V. et. al. 1996, Phys. Rev. D53 1314.
2. Barr G. et. al. 2003, (this Conference).
3. Battistoni G. et. al. 2000, Astropart. Phys. 12 315.
4. Honda M. et. al. 2001, Phys. Rev. D64 053011.
5. MACRO collaboration 1998, Phys. Lett. B434 451.
6. Lipari P. 2000, Astropart. Phys. 14 153.
7. Soudan-2 Collaboration 1999, Phys. Lett. B449 137.
8. SuperKamiokande Collaboration 1998, Phys. Rev. Lett. 81 1562.
9. Tserkovnyak V. et. al. 2003, Astropart. Phys. 18 449.