Atmospheric Muon Measurements at Sea Level II: A Maximum Likelihood Analysis

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Abstract

The Okayama telescope is an atmospheric muon detector with a solid iron magnet spectrometer. The muon momentum values measured have uncertainties due to multiple Coulomb scattering and the limited resolution of position chambers. We report the reduction of the uncertainties by exploiting a maximum likelihood technique. This technique would be applicable for other detectors.

1. Introduction

The Okayama telescope is an atmospheric muon detector with a solid iron magnet spectrometer [3,2]. Though muon momentum is estimated from the deflection angle in the magnet field, muons are deflected not only by the Lorentz force but also by multiple Coulomb scattering in the iron magnet. Thus, the measured muon momentum deviates from its true value. Furthermore, the limited resolution of "position chambers" (PC) also causes the deviation.

Various numerical deconvolution techniques may be helpful for some cases as the momentum spectrum measurement, but for event by event basis analyses. Hence, the reduction of the deviation in the momentum estimation is important. To attain the reduction, we use a maximum likelihood technique [1] to extract some information about the momentum from the nature of multiple Coulomb scattering.

Throughout this paper, the multiple Coulomb scattering distribution is approximated by the Gaussian distribution and we assume that the muon energy loss in the iron magnet is negligible and all angles concerned are small, that is, $\sin\theta = \tan\theta = \theta, \cos\theta = 1$.

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2. Likelihood Function

Fig. 1. shows a schematic view of a solid iron magnet spectrometer. The shaded rectangular region shows the iron magnet and the direction of the magnet field is perpendicular to the paper. The trajectory of a penetrating particle is estimated by using position chambers located above and below the magnet, PC1 and PC2. The trajectory projected to the z-x plane can be characterized by two straight lines as

$$x_1(z) = \hat{l}_{1,x} + \hat{\theta}_{1,x} z \quad (1)$$
$$x_2(z) = \hat{l}_{2,x} + \hat{\theta}_{2,x} z, \quad (2)$$

where eq. (1) and (2) are determined with PC1 and PC2 respectively.

Usually, the momentum of a incident particle is estimated only from the observed deflection angle $\hat{\theta}_x$ in the magnet,

$$\hat{\theta}_{\mathrm{mag}} = \hat{\theta}_x = \hat{\theta}_{2,x} - \hat{\theta}_{1,x}. \quad (3)$$

However, the particle trajectory projected to the z-y plane, that is, the observed deflection angle $\hat{\theta}_y$ also has some information about the particle momentum because the amount of multiple Coulomb scattering depends on the momentum. Both the expected deflection angle due to the magnet field and the root mean square (RMS) angle due to multiple Coulomb scattering are proportional to the inverse of the momentum, we can write

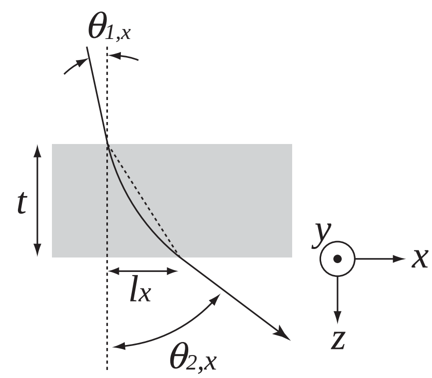

Fig. 1. A schematic view of a solid iron magnet spectrometer.

$$\sigma_{\theta_x,\mathrm{MS}}(\theta_x) = \sigma_{\theta_y,\mathrm{MS}}(\theta_x) = a\theta_x \quad (4)$$

where the proportional constant a is determined from the strength of the magnetic field and the thickness of the iron magnet t. Furthermore, the lateral displacement l_x and l_x give us additional information as

$$\frac{l_x}{t} = \frac{1}{2}\theta_x + \theta_{1,x}. \quad (5)$$

and the RMS value of l_y depends on the particle momentum.

Using the four observed values, θ_x, l_x, θ_y and l_y, we have the likelihood function $L(\theta_{\mathrm{mag}}|\theta_x, l_x, \theta_y, l_y)$ for the deflection angle due to the magnetic field θ_{mag} as,

$$
\begin{aligned}
&L(\theta_{\text{mag}}|\theta_x, l_x, \theta_y, l_y) \\
&\quad = \frac{1}{4\pi^2\sqrt{1-\rho_x^2(\theta_{\text{mag}})}\sqrt{1-\rho_y^2(\theta_{\text{mag}})}\sigma_{\theta_x}(\theta_{\text{mag}})\sigma_{l_x}(\theta_{\text{mag}})\sigma_{\theta_y}(\theta_{\text{mag}})\sigma_{l_y}(\theta_{\text{mag}})} \\
&\quad \times \exp\Bigg[-\frac{1}{2(1-\rho_x^2(\theta_{\text{mag}}))}\Bigg\{\frac{(\theta_x-\theta_{\text{mag}})^2}{\sigma_{\theta_x}^2(\theta_{\text{mag}})} + \frac{(l_x - E_{l_x}(\theta_{\text{mag}}))^2}{\sigma_{l_x}^2(\theta_{\text{mag}})} \\
&\qquad - \frac{2\rho_x(\theta_{\text{mag}})(\theta_x-\theta_{\text{mag}})(l_x - E_{l_x}(\theta_{\text{mag}}))}{\sigma_{\theta_x}(\theta_{\text{mag}})\sigma_{l_x}(\theta_{\text{mag}})}\Bigg\} \\
&\qquad - \frac{1}{2(1-\rho_y^2(\theta_{\text{mag}}))}\Bigg\{\frac{\theta_y^2}{\sigma_{\theta_y}^2(\theta_{\text{mag}})} + \frac{l_y^2}{\sigma_{l_y}^2(\theta_{\text{mag}})} - \frac{2\rho_y(\theta_{\text{mag}})\theta_y l_y}{\sigma_{\theta_y}(\theta_{\text{mag}})\sigma_{l_y}(\theta_{\text{mag}})}\Bigg\}\Bigg] \quad (6)
\end{aligned}
$$

where $E_{l_x}(\theta_{\text{mag}})$ denotes the expected value for l_x, $\rho_i(\theta_{\text{mag}})$ is the correlation coefficient between θ_i and l_i $(i = x, y)$, $\sigma_j^2(\theta_{\text{mag}})$ is $\sigma_j^2(\theta_{\text{mag}}) = \sigma_{j,\text{MS}}^2(\theta_{\text{mag}}) + \sigma_{j,\text{res}}^2$ $(j = \theta_x, l_x, \theta_y, l_y)$ and $\sigma_{j,\text{res}}$ is the RMS angle due to the PC resolution.

3. Monte Carlo Results

We performed Monte Carlo simulations to test the efficiency of our maximum likelihood estimator. For simplicity, we consider only the case that the deviation in the momentum estimation is dominated by multiple Coulomb scattering, $\sigma_{\text{MS}}^2 \gg \sigma_{\text{res}}^2$ and $\rho_x = \rho_y = \sqrt{3}/2$.

Fig. 2. shows the density distribution of $\hat{\theta}_{\text{mag}}$ for the case $a = \sigma_{\text{MS}}/\theta_{\text{mag}} = 0.3$. We made bias correction for the maximum likelihood estimates. (Maximum likelihood estimators are, in general, biased.) It is seen that the use of the maximum likelihood method narrows the width of the distribution.

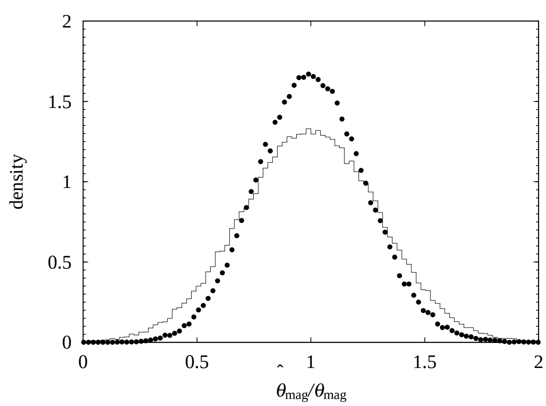

Fig. 2. The density distribution of $\hat{\theta}_{\text{mag}}$ obtained by eq. (3) (histogram) and the maximum likelihood method (full circle) for the case $a = 0.3$.

Fig. 3. and 4. shows the width of $\hat{\theta}_{\rm mag}$ distribution obtained by the maximum likelihood method as a function of a and the ratio of the width obtained by the maximum likelihood method to the one obtained by eq. (3) respectively. These figures indicate that our method is more effective in worse experimental condition (larger a).

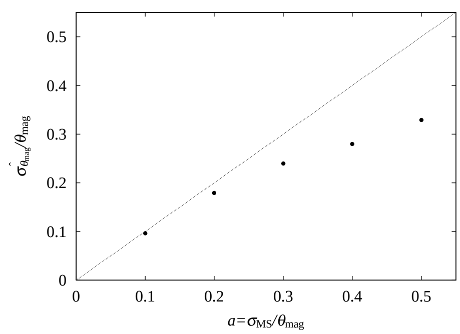

Fig. 3. The width of $\hat{\theta}_{\rm mag}$ distribution obtained by the maximum likelihood method (full circle).

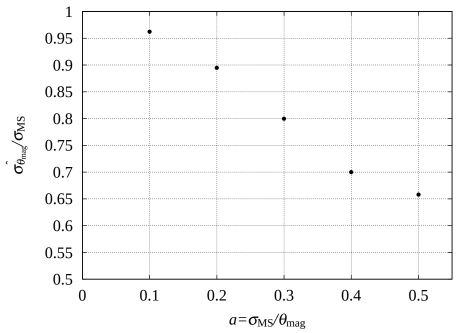

Fig. 4. The ratio of the width obtained by the maximum likelihood method to the one obtained by eq. (3).

4. Conclusions

We have developed a maximum likelihood technique to obtain better estimates of the muon deflection angles in the solid iron magnet spectrometer. The efficiency of our maximum likelihood estimator is investigated by Monte Carlo simulations. It is found that the RMS value of the estimated deflection angle can be decreased by applying our method. The amount of decrease is about 5, 10, 20, 30 and 35 % for $a = 0.1$, 0.2, 0.3 ,0.4 and 0.5 respectively. We therefore conclude that our maximum likelihood estimator is effective to analyze data obtained with solid iron magnet spectrometers.

References

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