Photon Yields from Dry Air Excited by Electrons

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Abstract

Recently we have reported the pressure dependences of photon yields and life times of the excited states for radiation in nitrogen and dry air by using a ^{90}Sr β-ray source [1]. In this report we mainly describe a parameters fitting and a separation method for superposition of two lines in one filter band.

1. Introduction

The photon yields from dry air excited by electrons in the troposphere are fundamentally important for estimating the primary energy of ultrahigh-energy cosmic rays (UHECR) by the fluorescence technique. An experiment has been undertaken using a ^{90}Sr β source to study the pressure dependence of photon yields and the life times of the excited states, for radiation in nitrogen and dry air. The results are submitted to Astroparticle Physics and can be referred to web-site [1].

We used six filters, named 316, 337, 358, 380, 391 and 400nm to represent the main fluorescence line contained in each filter of which width was about 10nm. All filters except 391nm include two or three lines from 2P bound from nitrogen molecule. While the 391nm filter transmits 391.4nm from 1N band of its ion which is significant, in addition to two 2P lines. So, for the formers we try parameters fitteing in terms of one component and for the latter of two components. Here we report photon yield from dry air.

2. One Component Analysis in a Filter Band

The photon yield ϵ_i for air through i-th filter band can be written as a function of pressure p by

$$\epsilon_i = \frac{C_i p}{1 + \frac{p}{p'_i}} , \tag{1}$$

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where C_i and p'_i are parameters which must be determined from experiments[1][2]. The fluorescence decay time τ is related to p'_i and another parameter τ_{oi} such as

$$\frac{1}{\tau} = \left(\frac{1}{\tau_{oi}p'_i}\right)p + \frac{1}{\tau_{oi}} \ . \tag{2}$$

As an example, in Fig.1 the p dependences of ϵ and of $\frac{1}{\tau}$ is shown in the case of 358nm band. We try to find two parameters to fit the $p-\epsilon$ diagram at first and estimate their errors . Since Eq.(5) is a hypabolic type function with respect to p, we take the following procedure. For each i-th band, first χ_1^2 is taken by weighted χ^2 with respect to ϵ. By minimizing χ_1^2, we find the first values of $(\frac{1}{p'})^{(1)}$ and $C^{(1)}$. Then after linearizing Eq.(5) using $(\frac{1}{p'})^{(1)}$ and $C^{(1)}$, Newton's method is applied to estimate the final $(\frac{1}{p'})^{(\nu)}$ and $C^{(\nu)}$ after the ν-th iteration. The statistical errors of $p'^{(\nu)}$ and $C^{(\nu)}$ are evaluated using the propagation law of errors. For a given $p'^{(\nu)}$, to find τ_0 and its error by the LS method is easy because Eq.(2) is linear with respect to $(\frac{p}{p'}+1)$.

The obtained results are shown by solid curves in Fig.1. For other filter bands except 391nm, three parameters with the errors are also obtained.

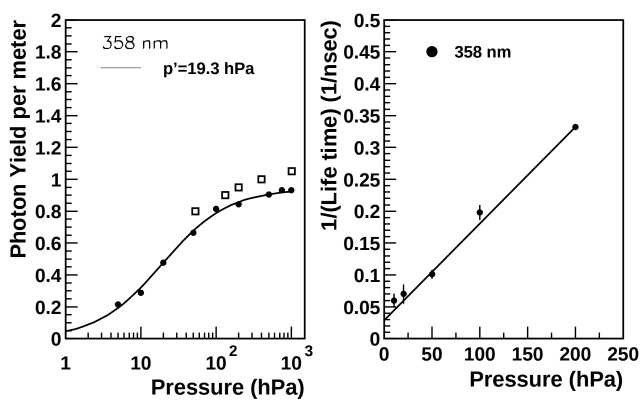

Fig. 1. An example of one line fitting in 358nm band in air. The data of Kakimoto et al.[3] is plotted by open squares. A solid line in the left figure shows the best fit of Eq.(1) with $p' = 19.3 \pm 1.4$ (hPa), C=4.86±0.27($\times 10^{-2}$/(hPa·m)) and that in the right one the best fit of Eq.(2) with $\tau_0 = 34.2 \pm 0.4$ (ns).

3. Two components analysis in one filter band

There are some discrepancies between the experimental points and the pressure dependencies of $\frac{1}{\tau_i}$ and ϵ_i in Eqs.(5) and (2) , especially in the 391 nm filter band. In the following, we try to fit the observed pressure dependencies of

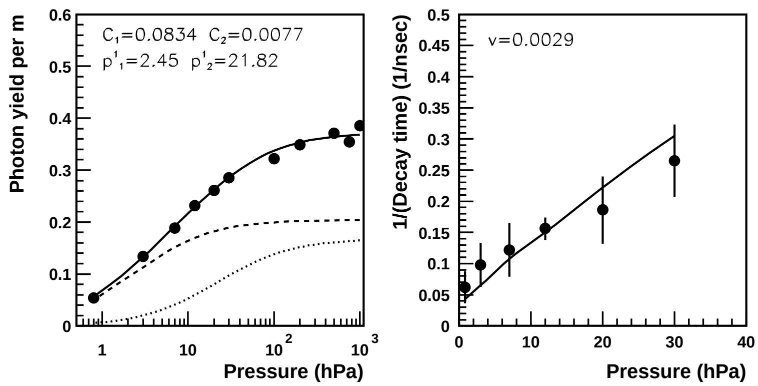

Fig. 2. Two line fitting of the 391 nm band in air. The contribution from the main line is indicated by a dashed curve, with the second line contribution indicated by a dotted line. A solid curve is the sum of the two lines in left-hand figures, and the solid line is Eq.(4) with v listed in the right-hand figures.

ϵ_i and $\frac{1}{\tau_i}$ with a superposition of two lines from 1N and 2P in one filter band. In this case the observed photon yield $\epsilon_{\rm obs}(p)$ is the sum of the photon yields of the main line (from 1N state) $\epsilon_1(p)$ and the sub-line (from 2P state) $\epsilon_2(p)$, and is written by extending Eq.(5) as follows:

$$\epsilon_{\rm obs}(p) = \epsilon_1(p) + \epsilon_2(p) \;\; = \;\; \frac{C_1 p}{1 + \frac{p}{p'_1}} + \frac{C_2 p}{1 + \frac{p}{p'_2}} \;\; , \tag{3}$$

where C_1 and p'_1 are parameters of the main line, and C_2 and p'_2 are parameters of another line. It should be noted that $\epsilon_{\rm obs}$ is determined assuming the filter transmission of the main line in the filter, and hence ϵ_2 must be corrected with using the filter transmission of that line.

The reciprocal of the observed life time $\frac{1}{\tau_{\rm obs}(p)}$ is approximately expressed by the weighted mean of $\frac{1}{\tau_1(p)}$ and $\frac{1}{\tau_2(p)}$. The weights are expressed in terms of the relative photon intensities of two lines. Then,

$$\frac{1}{\tau_{\rm obs}(p)} = \frac{p}{\epsilon_1(p) + \epsilon_2(p)} \times v \; , \quad \text{where} \quad v = \frac{C_1}{\tau_{o1}} + \frac{C_2}{\tau_{o2}} \;\; . \tag{4}$$

That is, τ_{o1} and τ_{o2} can't be evaluated independently without any assumptions and only the value v is determined by the LS method. In Eq.(3), we have to determine a set of four parameters , i.e. $\frac{1}{p'_1}, C_1, \frac{1}{p'_2}$ and C_2. For many sets, the value of χ^2 is computed and the set having the minimum χ^2 is taken as the initial set. Newton's method is also applied for this set, which leads to the final set after the ν-th iteration. Values of $\frac{1}{p'_1}, C_1, \frac{1}{p'_2}$ and C_2 are determined in this way.

The derivation of errors in this case is complicated in the usual LS method. So the errors of four parameters are estimated in the following way. Since the probability density function (pdf) of $\Delta\chi^2$, i.e. the deviation from $\chi^2_{\rm min}$, follows the pdf of χ^2 with four degrees of freedom, we take $\Delta\chi^2_{\rm critical}$=4.72 where the coverage probability is 0.683 . Many sets of (p''_1, C'_1, p''_2, C'_2) near to (p'_1, C_1, p'_2, C_2) are selected such that the value of χ^2 almost equals $\chi^2_{\rm min} + \Delta\chi^2_{\rm critical}$. The maximum and minimum values of $(p''_1 - p'_1)$ lead to the error bounds of p'_1. Errors of other three parameters are estimated in the same way. For v, its value and error are computed by the LS method.

4. Main Results on Photon Yields from Dry Air

Since p' for the 2P lines are nearly the same, the pressure dependence of total ϵ can be approximated as a superposition of two sets of terms, for the 2P lines and the 1N line, as follows:

$$\epsilon = \frac{C_{2P}\ p}{1+\frac{p}{p'_{2P}}} + \frac{C_{1N}\ p}{1+\frac{p}{p'_{1N}}}\ , \tag{5}$$

where $C_{2P} = 17.3 \pm 0.7$ ($\times 10^{-2}$/(hPa·m)), $p'_{2P} = 20.8 \pm 1.6$ (hPa), $C_{1N} = 1.33 \pm 0.23$ ($\times 10^{-2}$/(hPa·m)) and $p'_{1N} = 2.45 \pm 0.85$ (hPa). Here, the photons in wave bands not measured in this experiment, though thes photons are minor parts, are estimated from the list in Bunner[2] and the values are included in C_{2P}. The photon yield between 300 nm and 406 nm at 1000 hPa and 20 °C is 3.73±0.15 per meter for an electron of 0.85 MeV. The photon yield is proportional to $\frac{{\rm d}E}{{\rm d}x}$, and its density and temperature dependence of each line is tabulated in reference [1].

Since 391nm line is 1N band of the nitrogen ion and its life time is quite short compared to 2P lines, the contribution of this line to the total photon yields increases with altitude. its fraction to total is still less than 10% at altitude of 0.1 atmospheric pressure and seems to be negligibly small. However, this line is quite important for the distant showers from the observation point, if we take into account the attenuation of photons due to Rayleigh and Mie scattering.

5. References

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