Using Fractal Dimensionality in the Search for Anisotropy of Ultra-High Energy Cosmic Rays

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Abstract

Although the existence of cosmic rays with energies extending well above 10^{19} eV has been confirmed, their origin remains one of the most important questions in particle astrophysics today. Several different types of anisotropy have been proposed for the observed set of ultra high energy cosmic rays. Yet none of these models have been conclusively identified as corresponding with all of the available data. We propose a method by which one can use fractal dimensionality analysis to make a global measurement that would indicate the presence of *any* potential anisotropy. We will present the application of this analysis to the HiRes-1 monocular data at ICRC 2003

1. Introduction

The observation of Ultra-High Energy Cosmic Rays (UHECRs) has now spanned nearly three decades. Over that period, many different types of anisotropy have been proposed to explain the origin of these remarkable events. Recently, the Akeno Giant Air Shower Array (AGASA) reported clustering at small angular scales for the events that were observed above 4×10^{19} eV [9]. However, this result could not be confirmed by the High Resolution Fly's Eye (HiRes) air fluorescence detector [2] despite the fact that HiRes-1's monocular aperture was more than twice that of AGASA within the pertinent energy range [5]. HiRes also reported that it did *not* see anisotropies when examining harmonics in right ascension, *a priori* determined point sources or enhancement in the supergalactic plane. Furthermore, a prior analysis by the original Fly's Eye showed no evidence of any anistropies when dependencies in galactic latitude and longitude, harmonics in right ascension, excess maps, and specific *a priori* determined point sources were examined [3,4]. In 1995, it was reported by Stanev *et al.* that the combined data of Haverah Park, Yakutsk, and AGASA showed an excess along the supergalactic plane with several potential point sources for events above 2×10^{19} eV [8].

All of these conflicting results bring up a very pertinent question: Is there a more global way in which one could determine if a given sample possesses *any* statistically significant anisotropy? We will show that by considering the

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information dimension (i.e. the "entropy" of the distribution of arrival directions) of a given sample, one can simultaneously look for anisotropies at all angular scales greater than the angular resolution of the sample. This method is extremely robust, in that it can easily accomodate both asymmetric angular resolutions and asymmetric apertures. Furthermore, in the event that a data sample is shown to be consistent with an isotropic distribution, the same method can be used to place upper limits on possible anisotropic source models. Because only a single measurement is taken of the actual data, any number of potential anisotropies can be considered without incurring statistical penalties.

2. Calculating the Information Dimension of a Data Sample

Fractal dimensionality is a simple measure of a structure's scaling symmetry. By measuring the fractal dimension of a data sample, one can examine its self-consistency at different levels of magnification. There are several ways of going about this. From a computational perspective, the simplest way to examine fractal dimensionality is to use *box-counting*. For most general case, the *capacity dimension*, D_{C} [6], one partitions one's sample into equi-sized and equi-shaped "boxes" with edge size ϵ. D_{C} is then determined by considering the number of boxes that actually possess data.

However, the capacity dimension has a very severe limitation: It only looks for the *presence* of the sample within the available space, it does *not* consider variations in the *density* of the sample at a given point in the data space. In cases where the density may differ within the sample space, one can use the *information dimension* D_{I} [1,7]. D_{I} is essentially a measurement of the "entropy" of a set of data points. For a set of data points embedded on an n-dimensional manifold, a completely homogeneous data sample will yield $D_{\mathrm{I}} = n$. Any heterogenity in the data sample will lead to smaller values of D_{I}.

3. Application to Arrival Direction Distributions for UHECRs

In principle, it is simple to calculate the information dimension for a given sample of data points. However, there are a few complications that arise when one considers a set of arrival directions of UHECRs. First of all, the arrival directions are not known with complete precision. This makes the determination of D_{I} for $\epsilon \to 0$ meaningless. Secondly, the determination of D_{I} requires that the sample space be divided into equi-sized and *equi-shaped* bins. For a spherical surface, this is strictly not possible. However, there are workable solutions for both of these problems.

3.1. Angular Resolution

The angular resolution of monocular events observed by the HiRes-1 detector consists of two asymmetric orthogonal gaussian terms. By considering large sets of simulated events, one define these terms as follows: σ_1, σ_2:

$$\sigma_1 = 20^\circ e^{-1.5 \log_{10} E_{\mathrm{EeV}}} + 4.5^\circ \tag{1}$$

$$\sigma_2 = 100^\circ e^{-0.5 \Delta\chi} + 0.4^\circ \tag{2}$$

where E_{EeV} is the primary energy of the shower in EeV and $\Delta\chi$ is the angular track length (in degrees) of the shower as observed by the detector.

For the purpose of calculating D_{I}, we can treat the arrival direction of each individual shower as an elliptical gaussian distribution with the parameters σ_1, σ_2. Each shower direction distribution will have N_{Dist} points. The bin size, ϵ, will correspond to the scale length of the angular resolution of the sample ($\epsilon \simeq 0.5^\circ$).

3.2. Latitudinal Binning

For the purpose of calculating D_{I}, it also is necessary that all bins be equi-sized and equi-shaped as we vary the value of ϵ. While it is impossible to completely acheive this criterion on the surface of a sphere, we can to approximately do so by adopting a latitudinal binning scheme.

Latitudinal binning is achieved by first dividing the sky into N_δ declinational bands where each band has a width

$$\Delta\theta = \frac{\pi}{N_\delta} \tag{3}$$

We then determine that for each declinational band, the sky will be divided into $N_{\mathrm{RA},\delta}$ bins in right ascension where:

$$N_{\mathrm{RA},\delta} = \left[\frac{2\pi \int_{\delta 1}^{\delta 2} \cos\delta \, d\delta}{(\Delta\theta)^2}\right] = \left[\frac{2(N_\delta)^2 \int_{\delta 1}^{\delta 2} \cos\delta \, d\delta}{\pi}\right] \tag{4}$$

Hence the solid angle, $\Delta\Omega_\delta$, of each bin (in steradians) is:

$$\Delta\Omega_\delta = \frac{2\pi \int_{\delta 1}^{\delta 2} \cos\delta \, d\delta}{N_{\mathrm{RA},\delta}} \tag{5}$$

with a minimum value of $(\Delta\theta)^2$ (at the equator) and a maximum value of $\frac{\pi}{3}(\Delta\theta)^2$ (at the poles) regardless of the value of N_δ. This provides us with bins that are all almost the same area and nearly square-shaped (with the exception of three triangular bins at each pole). The total number of bins in the sky can be approximated by:

$$N_{sky} \simeq 4\pi (\frac{N_\delta}{\pi})^2 = \frac{4}{\pi}(N_\delta)^2 \tag{6}$$

4. Results

By comparing the calculated values of D_{I} for real and simulated sets, we assess the probability that our event sample conforms with a given anisotropic source model. Because we are relying on a measurement of a *single* parameter, we will be able to consider any number of potential anisotropic source models simultaneously without depleting the statistical significance of our findings. We will present the application of this analysis on the HiRes-1 monocular data at ICRC 2003.

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