
Internal Structure of Ultra-High Energy Particles with Lorentz Symmetry Violation at the Planck Scale

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Abstract

Assuming the existence of a local vacuum rest frame (LVRF), and using suitable algebraic transformations, the internal structure of ultra-high energy particles (UHEPs) is studied in the presence of Lorentz symmetry violation (LSV) at the Planck scale. Violations of the standard Lorentz contraction and time dilation formulae are made explicit. Dynamics in the rest frame of a UHEP is worked out and discussed. Phenomenological implications for ultra-high energy cosmic rays (UHECR), including possible violations of the Greisen-Zatsepin-Kuzmin (GZK) cutoff, are studied for several LSV models.

1. Models of Lorentz Symmetry Violation

In the period 1995-97, we suggested several patterns of Lorentz symmetry violation [4-7], all of them leading (for the "ordinary" sector of matter, i.e. that with critical speed in vacuum equal or very close to the speed of light) to deformed Lorentz symmetries (DLS) and to deformed relativistic kinematics (DRK). Such LSV patterns were further discussed in subsequent papers, f.i. [8-13], especially quadratically deformed relativistic kinematics (QDRK) where for $k a \ll 1$ ($k =$ wave vector, $a =$ fundamental length) the particle energy E can be written as:

$$E \simeq (2\pi)^{-1} h c a^{-1} [(k a)^2 - \alpha (k a)^4 + (2\pi a)^2 h^{-2} m^2 c^2]^{1/2} \quad (1)$$

h being the Planck constant and c the speed of light. α is a model-dependent constant that may be in the range 0.1 – 0.01 for full-strength violation of Lorentz symmetry at the fundamental length scale, and m the mass of the particle. For momentum $p \gg mc$, $E - [p c + m^2 c^3 (2 p)^{-1}] = \Delta E \simeq - p c \alpha (k a)^2 / 2$. It is assumed that the earth moves slowly with respect to the absolute rest frame. For physical reasons developed in [14,15], we discarded linearly deformed relativistic kinematics (LDRK) which in the region $k a \ll 1$ is given by:

$$E \simeq [(k a)^2 - \beta (k a)^3 + (2\pi a)^2 h^{-2} m^2 c^2]^{1/2} \quad (2)$$

β being a model-dependent constant. For $p \gg mc$, $\Delta E \simeq - p c \beta (k a) / 2$ which is often invoked to explain *TeV* gamma data [1,2].

The analysis of these models, from the LSV point of view, was updated in [14-16] and has been further developed in [17,18]. LSV patterns based on mixing with superluminal sectors of matter are discussed elsewhere in this Conference. More generally, we shall mainly discuss here LSV patterns based on the appearance of a fundamental length scale which can be identified with the Planck scale but can also be interpreted otherwise. However, the models we consider are different from the patterns proposed by Kirzhnits and Chechin [3,19,20] and by Sato and Tati [23], and should be compared with them to make differences clear.

The Kirzhnits-Chechin (KCh) model was based on a Finsler space. The relativistic relation $p_0^2 = p^2 c^2 + m^2 c^4$ (with $p^2 = \sum_{i=1}^3 p_i^2$, the i 's being space indexes) was replaced by: $f(p_\mu) (p_0^2 - p^2 c^2) = m^2 c^4$, $\mu = 0, 1, 2, 3$ standing for standard four-momentum indexes and $f(p_\alpha)$ being a homogeneous positive function of the four-momenta of zero degree. These authors used $f(p_\mu) = f(\xi)$ where $\xi = [p^2 c^2 (p_0^2 - p^2 c^2)^{-1}]$, $f(0) = 1$ and f was supposed to tend to some constant $f(\infty)$ in the range 0.01 - 0.1 as $\xi \rightarrow \infty$. This amounts to a shift of the effective squared mass by a factor 10 to 100 above some critical value of ξ . The dispersion relation for the photon was assumed to have no deformation, and f was taken to be $f(p_\mu) = f(\infty)$ for this particle. For a massive particle, it was assumed that f can be expanded as: $f(\xi) \simeq 1 - \kappa \xi^2 + \dots$ leading, to a first sight, to models close to QDRK. For the proton, the term $\kappa \xi^2$ becomes ≈ 1 at E_p (proton energy) $\approx 10^{20}$ eV if $\kappa \approx 10^{-44}$ (the *ad hoc* choice to fit data). In [18], we have worked out numerical examples and shown that actually the KCh pattern does not allow to build suitable models to explain the possible absence of GZK cutoff. But we also pointed out that, using an extension [17,18] of the Magueijo-Smolin operator formalism [21,22], the KCh pattern can be successfully modified and unified with the QDRK approach in a larger class of models, including patterns with extra space-time dimensions.

The Sato-Tati (ST) model was equally proposed as a solution to the UHECR puzzle. It implies the existence of a preferred reference frame and the impossibility for hadronic matter to exist above a value of the Lorentz factor $\simeq 10^{11}$ with respect to this frame. Contrary to DRK, this model involves a very strong dynamical assumption on the production of hadronic matter at ultra-high energy. Rather than with the structure of space-time, it seems to be concerned with the dynamical properties of vacuum in our Universe and with those of hadronic matter. Even with a privileged LVRF, the ST model can incorporate exact relativistic kinematics and have only a sharp dynamical threshold for the inhibition of hadronic particle production. Furthermore, as discussed in [18], the suppression itself of the GZK cutoff in the ST model is unclear. However, the question of whether hadronic matter can exist above some critical value of E/m in the LVRF is a fundamental one and certainly worth addressing. Some aspects of this problem are presently under study using the operator formalism developed in [17,18].

2. Internal Structure and Space-Time Issues

The idea of Doubly Special Relativity (see [1,2] and references therein) has led to the study of the possible formal equivalence between theories with Lorentz symmetry and theories with deformed Lorentz symmetry. This study led in turn to the suggestion by Magueijo and Smolin [21] to use an operator formalism to relate both kinds of theories. In recent papers [17, 18], we further developed this idea and suggested new ways and potentialities. In particular, the operator formalism can be used to go to the rest frame of a UHEP and study the dynamics as seen by the particle. This would allow to examine, for the first time, fundamental dynamics such as it is seen by elementary particles in a rest frame corresponding in the LVRF to a momentum scale closer, in logarithmic scale, to Planck scale than to the electroweak scale (e.g. for protons above $E \approx 10^{19} eV$). There is no fundamental reason for the laws of Physics at these scales to look like we imagine them from laboratory formulations. There is even no compelling evidence that quantum mechanics is not violated for UHEPs together with special relativity. To study these crucial questions, the analysis of UHECR data may provide a powerful microscope directly focused on Planck scale dynamics.

A simple illustration was provided long ago using simplified soliton models. In a model using an analogy with the one-dimensional Bravais lattice [7,9,10], it was shown that nonlocal effects at the $\approx a$ scale can change the internal structure of a relativistic object at distance scales well above a (see also Gonzalez-Mestres, this Conference). With an example leading to QDRK, it was shown that if the typical size of a relativistic soliton is $\gamma^{-1} \Delta$, γ being the effective Lorentz factor and Δ a characteristic distance scale from soliton dynamics, the effective inverse squared Lorentz factor γ^{-2} is corrected by a power series of ξ , $\gamma^{-2} = \gamma_R^{-2} + \gamma' \xi + \dots$, γ' being a constant of order 1. Then, we expect the departure from standard relativity to play a leading role at energies above that for which $\gamma_R^{-2} \approx \alpha (a \gamma_R)^2 \Delta^{-2}$, i.e. above $E \approx m c^2 \alpha^{1/4} (a \Delta^{-1})^{-1/2}$. Taking the values $\alpha \approx 0.1$, $m \approx 1 GeV/c^2$ and $\Delta \approx 10^{-13} cm$, this energy scale corresponds to E above $\approx 2 \cdot 10^{19} eV$ for $a \approx 10^{-33} cm$. Therefore, the internal structure of the UHEP changes drastically above this energy. That this is indeed the case can be checked [17,18] using more recent techniques where an operator formalism allows to go to the rest frame of the UHEP.

To roughly illustrate, without explicitly using operators, how a DRK boost technique can work, assume that in the LVRF a particle satisfies the QDRK:

$$p_0^2 = p^2 + m^2 - b p^4 \quad (3)$$

where b is a constant, $b m^2 \ll 1$ and we have taken $c = 1$. We can write the deformation term as: $b p^4 = b (\pi_\mu \pi^\mu)^2$ with $\pi_\mu = p_\mu - V^{-2} (p_\mu V^\mu) V^\mu$, V^μ being a quadrivector with value $(V, 0, 0, 0)$ in the LVRF characterizing an apparent spontaneous breaking of Lorentz symmetry (SLSB) and V any constant.

We can then perform a Lorentz transformation with boost parameter Γ from the LVRF to the rest frame of a UHEP obeying (3), by writing:

$$p_0 + p_3 = \Gamma (p'_0 + p'_3) \quad (4)$$

$$p_0 - p_3 = \Gamma^{-1} (p'_0 - p'_3) \quad (5)$$

$$V = \Gamma (V'_0 + V'_3) = \Gamma^{-1} (V'_0 - V'_3) \quad (6)$$

where the p_μ quadrivector stands now for energy and momentum as measured in the LVRF, and p'_μ is the same quadrivector as measured in the rest frame of a UHEP of momentum p_Γ pointing in the direction of the spatial axis $i = 3$. Similar conventions hold for V_μ and V'_μ . Calculations show then that the equations of motion, as seen in the UHEP rest frame, present a singularity at $b p_\Gamma^4 = m^2$, i.e. when the deformation term becomes equal to the mass term (well below Planck scale). More details, using explicitly the operator formalism, can be found in references [17] and [18], as well as in subsequent papers of the same series (*Deformed Lorentz Symmetry and High-Energy Astrophysics*, see arXiv.org).

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