On the Cherenkov Light Contribution to the Fluorescence of the Highest Energy Air Showers

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Abstract

In the experiments for detecting the highest energy air showers by the induced atmospheric fluorescence (Fly's Eye, Auger) it is important to know what is the contribution of the Cherenkov light to the total light emitted by the shower in different directions. Here we show that the effective fraction of electrons emitting Cherenkov light at a given level in the atmosphere depends only on the shower age and height of this level. It practically does not depend on the primary particle (neither on its energy nor mass). We give an analytical formula for it, fitted to the CORSIKA simulation results.

1. Introduction

It has been acknowledged that the contribution of the scattered Cherenkov light to the fluorescent one, emitted by a shower at large angles to the axis, is not to be neglected and has to be carefully evaluated [1]. The indispensable tool here is CORSIKA or any other extensive air shower program. Some authors propose to simulate shower with the Cherenkov light calculations included. We think that this would be an unnecessarily long way and propose here a much shorter and effective method to calculate the Cherenkov light produced by a shower.

One problem to be overcome is that not all electrons emit it, in contrast to the fluorescence light, where its amount produced at a given level in the atmosphere is practically proportional to the total number of electrons at this level. The electron threshold energy E_{thr} for the Cherenkov emission to occur decreases with depth X in the atmosphere as $\frac{1}{\sqrt{X}}$, reaching 23.7 MeV at the Auger level. Above the threshold energy it saturates rather quickly, reaching $\sim 90\%$ of its maximum value at $E = 3E_{thr}$. Thus, the effective fraction of electrons emitting Cherenkov at a given level, defined as the total number of Cherenkov photons produced in a small slice of the atmosphere at this level, divided by the maximum number of photons produced by one electron in this slice, should depend only on the shape of the electron spectrum and on E_{thr} at this level. Our aim here is to find this dependence.

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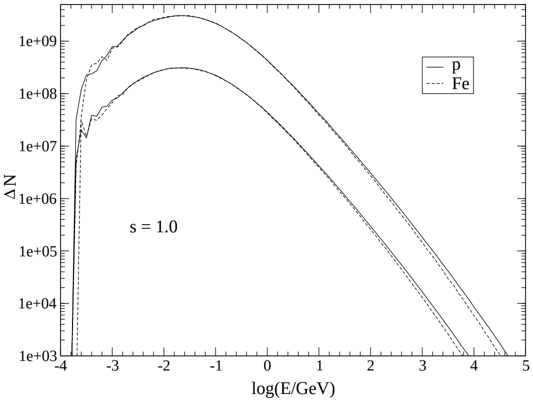

Fig. 1. Comparison of electron energy spectra at shower maximum ($s = 1$) for protons (solid line) and Fe (dashed line), for $E = 10^{19}$eV (bottom curves) and $E = 10^{20}$eV. ΔN is number of electrons in $\Delta logE = 0.1$. Each spectrum is an average from 10 showers.

2. Electron energy spectra

2.1. Dependence on the age parameter s

The shape of the electron energy spectrum at a given height should depend on the stage of the shower development. In the pure electromagnetic cascade theory it is the age parameter s, growing monotonically with the cascade development. We will show that it can also be used to describe the "age" of a hadronic shower, i.e. that the electron energy spectra can be described by the s parameter only, defined as : $s = 3X/(X + 2X_{max})$, where X is the atmospheric path (in $g \cdot cm^{-2}$) traversed by a shower, and X_{max} is the path to the shower maximum. Fig.1. shows energy spectra of electrons in proton and Fe initiated showers, both for primary energy $E_0 = 10^{19}$ and 10^{20} eV. For each case 10 showers were simulated by CORSIKA. For each shower the spectrum was found in the shower maximum and then the average calculated. It can be seen that the spectrum is the same independently of the primary particle. It has also practically the same shape for both primary energies.

Fig.2. illustrates that the s parameter, as defined above, describes the shapes of the energy spectra at other stages of shower development as well, independently of primary energy or mass. In this figure there are plotted ratios of the energy spectrum for $E_0 = 10^{20}$eV to that for $E_0 = 10^{19}$eV, for various values of s, for proton (left) and Fe (right) showers. It is seen that a deviation from the flat line occurs only at high electron energies, but it exceeds 10% for energies larger than 10^3 GeV where there are negligibly few particles. The scatter of points at low energies is caused artificially by the thinning method. Thus, *the shapes of energy spectra do depend on the age parameter s only.*

In the Auger experiment it is single showers that one will have to analyse and one could worry whether fluctuations from shower to shower could change the shape of the spectrum. We have checked that this is a small effect. The only difference can be seen at low energies, most probably caused mainly by thinning.

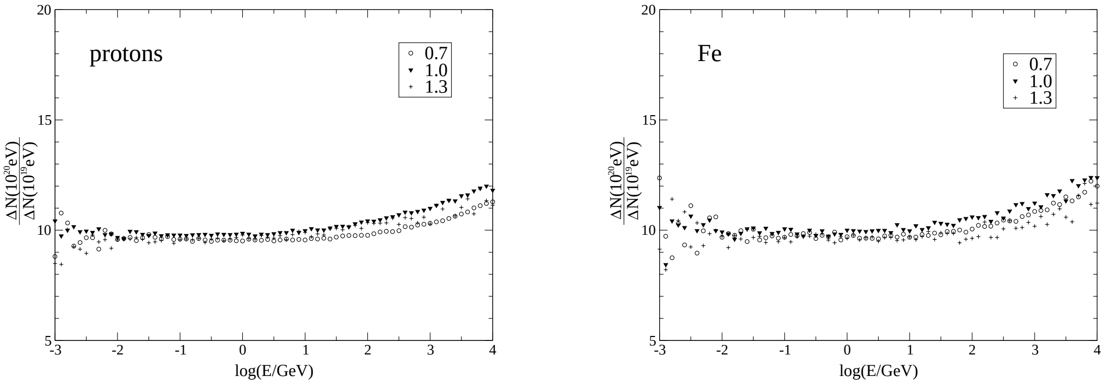

Fig. 2. Ratio of electron energy spectra for $E_0 = 10^{20}$eV to that for $E_0 = 10^{19}$eV at different stages of shower development ($s = 0.7$, 1, 1.3).

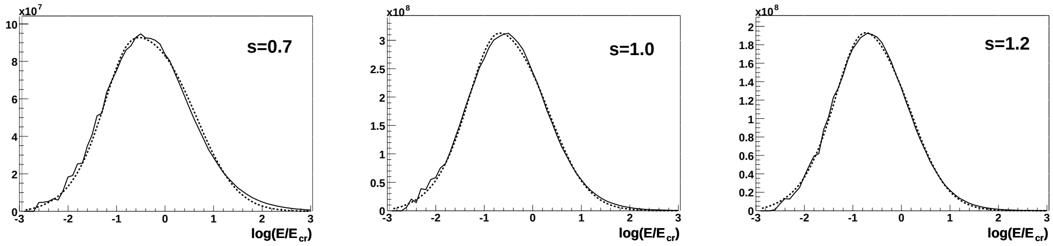

Fig. 3. Comparison of the calculated energy spectra (solid line) with the analytical formula (dotted line).

2.2. The analytical formula

A fit to the electron energy spectra in showers of much lower primary energies has been proposed by Hillas [2]. We have checked that it does not describe the CORSIKA results for $E_0 = 10^{19}-10^{20}$eV sufficiently well so that a new formula would be useful. We have fitted an analytical formula describing quite well the shapes of the electron energy spectra for different ages:

$$\frac{1}{N_0}\frac{dN_e}{dlogE} = C(s)\cdot\left\{1 - a\cdot\exp\left[-f(s)\cdot\frac{E}{E_{cr}}\right]\right\}\cdot\left(1+\frac{E}{E_{cr}}\right)^{-\left[s+b\cdot ln\left(\frac{E}{c\cdot E_{cr}}\right)\right]} \tag{1}$$

where $a = 1.015$, $b = 0.13$, $c = 60.6$, $f(s) = 6.84\cdot s + 11.84$ and $C(s) = 0.262\cdot s + 0.270$ in $0.7 \le s \le 1.3$ region.

We have adopted here $E_{cr} = 80$ MeV, as close to the critical energy of the air. N_0 is the total number of electrons at the level of age s.

The above formula has been fitted having in mind the analytical results of the electromagnetic cascade theory. The electron energy spectrum for $E \gg E_{cr}$ is $\frac{dN_e}{dlogE} \sim E^{-s}$ for a pure 1-dim. cascade, with the same definition of the shower parameter s. For a hadronic shower the spectrum is a bit concave (see Fig.1.),

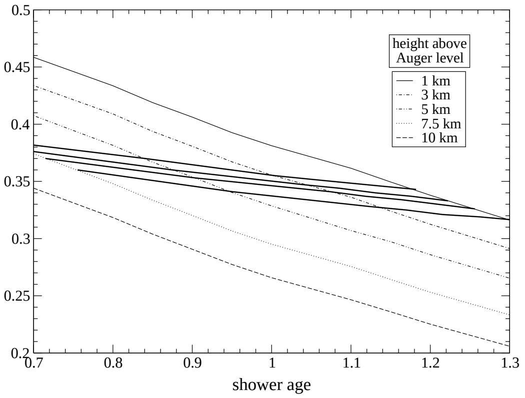

Fig. 4. Effective fraction of electrons $F(s,h)$ emitting Cherenkov light as a function of shower age and height above the Auger level. The thick, almost horizontal lines correspond to typically developing showers for four cases (from top to bottom): proton 10^{20}eV, proton 10^{19}eV, Fe 10^{20}eV and Fe 10^{19}eV.

so that to allow for this we have introduced a small term in the power index (constants b and c). For $E \sim E_{cr}$ and smaller the spectrum turns down and the factor in the formula responsible for this is that with the exponent. A comparison of the calculated spectra with the fitted formula is represented in Fig.3. In the region $s = 0.7 - 1.25$ the fit describes very well the electron spectra in the most important energy range $\frac{E}{E_{cr}} < 10^2$.

3. Fraction of electrons emitting Cherenkov light

The effective fraction of electrons emitting Cherenkov light at a given height in the atmosphere depends only on this height and the shower age s. We have calculated the dependence on both variables and the result is shown in Fig.4. It applies to all primary energies and masses. Thus, analysing a shower we have to know only the position of its maximum to be able to determine its age at any height and thus, the fraction of electrons $F(s,h)$ producing new Cherenkov photons. A useful analytical form fitted to the results is the following:

$$F(s,h) = 0.64 - 0.24 \cdot s - 0.0129 \cdot h(km) \tag{2}$$

where h is measured above the Auger level (1452 m). There is, however, a rather strong correlation between s and h, so that not all regions on Fig.4. are equally important. The actual fraction of "Cherenkov electrons" does not vary much along a shower, decreasing by less than 15%. The total Cherenkov light contribution (and that scattered) on a given level depends on the atmosphere properties on this level and above it, so it can not be treated further in a universal way.

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1. Cronin J. et al., 1997, Pierre Auger Design Report, http://www.auger.org
2. Hillas A.M. J. Phys. G: Nucl. Phys 8 (1982) 1461-1473