Analytical Time Structure of Muonic Showers

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Abstract

An analytical description of the time structure of the pulses induced by muons in air showers at ground level is described in terms of production distributions for distance, energy, and transverse momentum. The results of this model agree with those obtained with simulations. Major contributions to muon time delays are identified.

1. An analytical description of muon distributions in air showers

During the last decade many efforts have been made to study the high energy end of the cosmic ray spectrum. The largest air shower array, the Southern Auger Observatory, is now under construction [1]. The particle detectors of the Observatory have been designed to establish the time structure of the signal with unprecedented detail. The early part of the signal is dominated by muons that reach the ground with more energy than photons and electrons because they make fewer catastrophic interactions. Understanding the time structure of the signal is of crucial importance in establishing the capability of these detectors for reconstructing the arrival direction of the primary particles and for optimizing the information obtained from these detectors, in particular that from which mass information can be extracted. We have developed an analytical model that reproduces the time structure of the muons at ground level as obtained in simulation results which will prove to be a powerful tool for its study.

This model is based on the assumption that the probability distribution for muon production factorizes completely into a product of three independent distributions, the energy at production, E_0, the transverse momentum, p_t (both at production), and the distance z from the point of production to the ground

$$\frac{d^3N_\mu}{dp_t dE_0 dz} = f_1(p_t) f_2(E_0) G(z). \tag{1}$$

The muon distribution at production, $G(z) = dN/dz$, depends on the details of the hadronic model, on the primary energy, and the primary chemical composi-

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tion. We parameterize it from simulations using AIRES [4]. A sufficiently good approximation for $f_1(p_t)$ is simply inspired by hadronic interactions, $f_1(p_t) = Bp_t^\lambda \exp(-p_t/Q)$, and for $f_2(E_0)$ we can use a simple power law, $f_2(E_0) \equiv \frac{dN}{dE_0} = AE_0^{-\gamma}\Theta(E_0 - m)$, typical of showering processes.

To account for distributions at ground level muon energy loss and decay must be considered. This can be done analytically provided that a constant energy loss per unit length $dE/dl = -k\rho$ is assumed, ρ being the atmospheric density and k the energy loss per unit of grammage. With these ingredients it is relatively straight forward to obtain the muon energy distributions at ground level as a function of distance to shower axis r, using the simple geometrical relation $p_t/E_0 = r/l$ already used in [2]. (Details will be published elsewhere.) Fig. 1. shows the agreement between the model and the simulation for a proton shower of 80°. Eq. 1 ignores correlations between z, p_t, and E_0. These correlations can

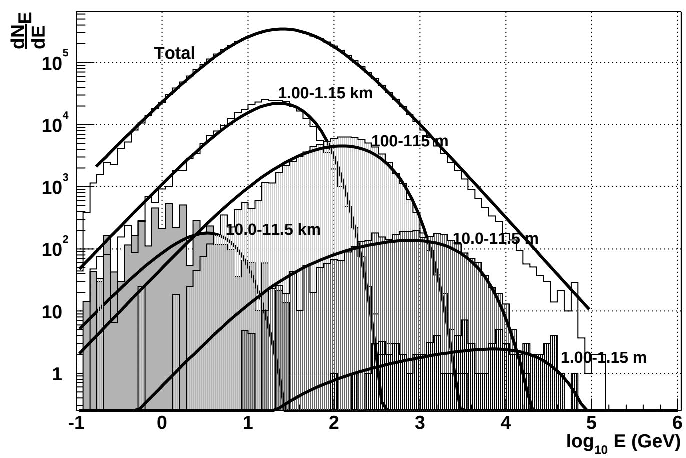

Fig. 1. Final energy spectrum at different r. Histograms show Monte Carlo results obtained with AIRES for a 10^{19} eV proton shower at 80° zenith angle compared to our prediction for the spectrum indicated by the continuous lines.

be shown to be very small compared to the width of the distributions. Our model also assumes straight line muon propagation, neglects the transverse distribution of the parent particles and ignores magnetic field effects. Simulations show that magnetic effects do not have very important implications up to zenith angles of 80° to 85° at the geomagnetic latitude of Malargüe, that multiple scattering is not very relevant for the bulk of muons, particularly in inclined showers, and that the effect of the transverse position of pions at decay is small compared to the typically large distances between particle detectors in extensive air shower arrays. As a result the simple model has a considerable predictive power.

2. Time Distributions

The total time delay of the arriving muons can be expressed as a sum of two delays $t = t_g + t_\epsilon$, where t_g is the geometrical delay time, due to path differences and t_ϵ is the kinematic delay due to muon velocity, slower than light. They have corresponding distributions $g(t)$ and $\varepsilon(t)$ which can be combined in a simple convolution to get the final time distribution

$$\frac{dN}{dt} = g(t) \otimes \varepsilon(t) = \int g(t - t')\varepsilon(t')dt'. \tag{2}$$

The geometrical delay with respect to a light particle traveling along shower axis is simply obtained from the path difference (see Fig.2.)

$$t_g = \frac{1}{c}\left[\sqrt{(z-\Delta)^2 + r^2} - (z - \Delta)\right]. \tag{3}$$

Here Δ gives the shift in z (measured along shower axis) corresponding to different locations on the ground. Using polar coordinates (r, ζ) in a plane transverse to shower axis $\Delta = r\cos\zeta\tan\theta$. The arrival time distribution of the muon signal at a transverse distance r relates to the distance distribution $G(z)$ through

$$g(t_g; z, r) \equiv -\frac{dN_\mu}{dt} = -\frac{dN_\mu}{dz}\frac{dz}{dt} = -G(z-\Delta)\frac{dz}{dt}. \tag{4}$$

Δ introduces the asymmetries that are found using simulations. Muons are also delayed because of their velocity β

$$t_\epsilon = \frac{1}{c}\int_0^l dl'(\frac{1}{\beta(E)} - 1), \tag{5}$$

which takes continuous energy loss into account through $\beta(E)$. If energy loss per unit length is constant this expression can be analytically integrated

$$t_\epsilon \simeq \frac{1}{2}\frac{m^2}{c\rho k}\left[\frac{1}{E_0 - \rho k l} - \frac{1}{E_0}\right]. \tag{6}$$

When the energy spectrum of the muons is known, the arrival time distribution due to this effect is then given by

$$\varepsilon(t_\epsilon; z, r) \equiv -\frac{d^2N_\mu}{dt_\epsilon dr} = -\frac{d^2N_\mu}{dE_0 dr}\frac{dE_0}{dt_\epsilon}. \tag{7}$$

We can now calculate the total time delay distributions: we substitute dE_0/dt and the energy spectrum into Eq. 7, a production distribution into Eq. 4 (which can be parameterized from simulation results) and we use the time convolution indicated in Eq. 2. The final result is cumbersome and we do not give it explicitly here.

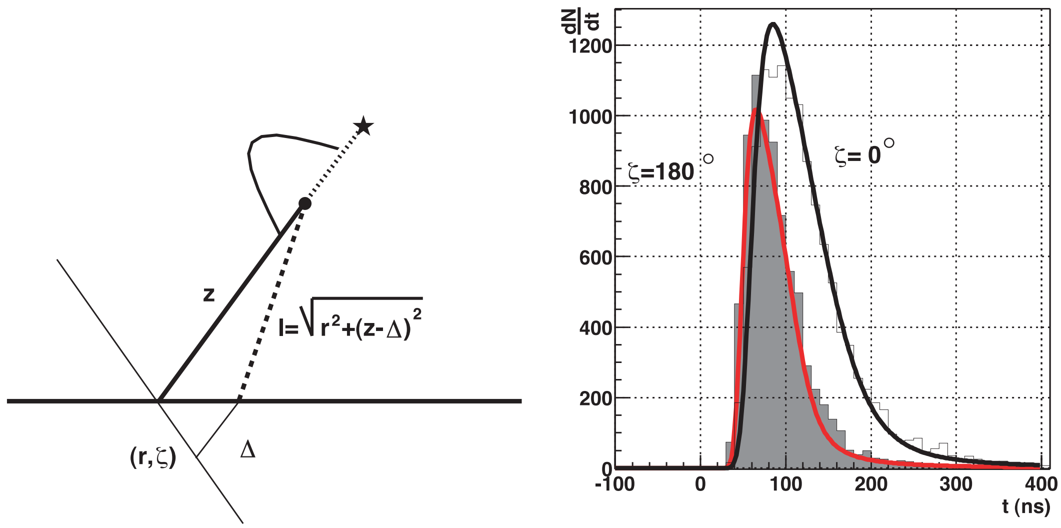

Fig. 2. Left pannel: Schematic diagram of the geometry and notation used for obtaining the arrival time distribution of muons in air showers. Right pannel: Analytical time distribution (lines), for a 80° proton shower at $r = 2000$ m, at 1400 m altitude, for two different polar angles compared to simulation results (histograms).

The results are extremely encouraging as illustrated by the comparison between the analytical and simulation time distributions shown in Fig. 2.b. The agreement has been checked for different zenith angles and transverse distances. The model agrees with simulations at least down to $\theta = 30°$ and for 100 m $< r <$ 4000 m . The model has allowed us to interpret that most of the arrival time distribution is due to geometrical delays and that kinematic effects are important at the 25% level. This is in agreement with [3]. The advantages of such a model are obvious since it can be used as a powerful tool in optimizing the information that can be obtained from the time distribution of signals in ground arrays. Much work on this line is now in progress and will be reported elsewhere.

Acknowledgments: This work is supported by Xunta de Galicia (CESGA and PGID T00PXI20615PR), by CICYT (AEN99-0589-C02-02), by MCYT (FPA 2001-3837 and "Ramón y Cajal" program) and by PPARC (PPA/Y/S/1999/00276).

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