Description of Cascades with Energies Above the GZK Cut-Off

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Abstract

It is consider results of Monte-Carlo simulation of air showers generated by protons of energy 10, 100, 1000 EeV for N(t) - the number of electron at dept t. The showers are described by random variable parameters of approximation of individual cascades. Distribution functions of the parameters are investigated. It is note insufficient description of the showers by mean values and simple dispersion only. Approximation formulas for mean values and complex dispersion are presented.

1. Introduction

The present paper uses results of Monte-Carlo simulation of air showers generated by protons of energy 10, 100, 1000 EeV for zenith angles of $\theta = 0°, 24°36', 44°24', 60°$ with statistics 500 for each cause. Calculation were carried out in the QGS model [3]. The hybrid method [2] was used for estimate both the mean values and the standard deviations of the physical parameters [1]. The obtained results were presented as functions on the atmospheric depth $t(g/cm^2)$ with step $\Delta t = 50 g/cm^2$. The technique quality of the simulation results was rather high: "jump-out" points and local unevennesses are rare and their deviations are less than the natural cascade fluctuations. The present paper considers the description of the number of electrons $N(t)$ in a shower at depth t.

The paper [4] had presented the method of accuracy (about 1.5 %) in interval $(t_m/2 \div 2t_m))$ approximation of an individual cascade by form of

$$N(t) = N_m exp(-(t-t_m)^2/(a(t-t_m)+2b^2)). \qquad (1)$$

Parameters (1) have clear geometric and physical interpretation: t_m, N_m are location of the maximum of the individual cascade, a is asymmetry parameter, $2b^2$ characterize width of the curve $N(t)$ as σ^2 for Gaussian distribution.

The approach presents each individual cascade by values of 4 parameters and vice versa, that is individual cascade is concrete realization of the 4 random

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variables, which probability density we denote: P_{tm}, P_{Nm}, P_a, P_b. From the other hand 4 values of parameters describe the individual cascade and the functions P_{tm}, P_{Nm}, P_a, P_b describe whole ensemble of the cascades complete. As soon as $\int_0^\infty N(t)dt \approx \sqrt{\pi} N_m \sqrt{b}$, parameter $c = N_m \sqrt{b}$ is useful most stability parameter with clear geometry and physical interpretation and it's probability density we denote P_c. This interpretation of the parameters enable to understand the main characteristics the distribution functions.

2. Distribution Function of the Cascades Parameters and Discussion

This paper describes the distribution functions of the parameters t_m, N_m, a, b, c by it's probability density functions. Fig. 1 shows normalized histograms and their approximations of P_{tm} and P_{Nm} by form (1). Definition of σ_l and σ_r see below.

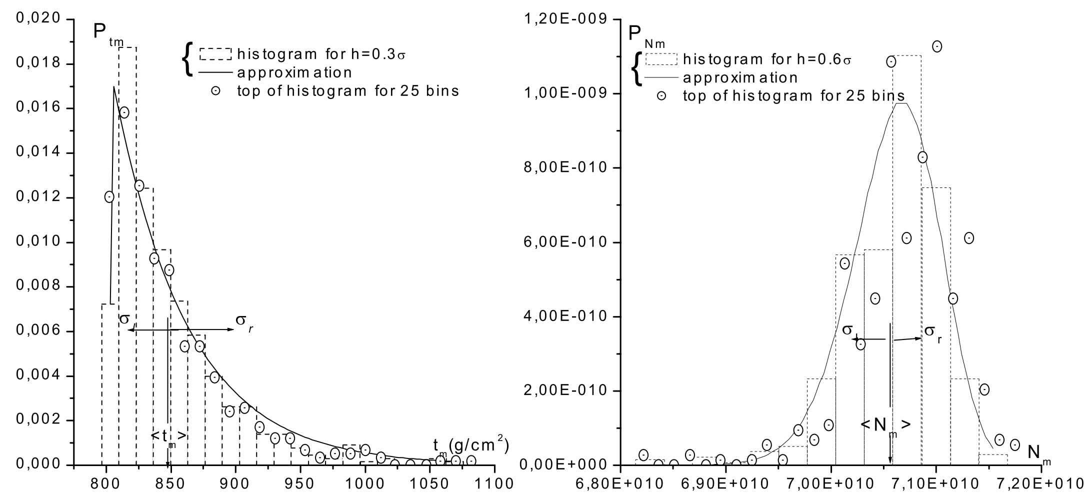

Fig. 1. Example of probability density function of the maximum t_m, N_m air showers generated by 500 protons of energy 100 EeV for zenith angle 60°.

The function P_{tm} for the parameter t_m shows a narrow peak shape at left side from $< t_m >$ - mean value of t_m and wide ($\approx 10\sigma$) range of the parameter with approximately exponential asymptotic behaviour. The depth of cascade maximum is determined by two physical processes: the depth of the first interaction (with the known - exponential distribution of the depth) and cascade development. Therefore the probability density function for t_m is also (as the exponent) strongly asymmetric to the left with the maximum not at the left edge but at a distance from the edge about σ , it is coming out of the cascade fluctuation.

The function P_{Nm} has the range of the parameter $\approx 7\sigma$, the mean value is shifted to right from middle and maximum is some shifted to right from mean value and has not so narrow peak shape as P_{tm}. The form of the distribution is determined by cascade fluctuation and co-ordinate with well- known form of relative variation ($VarN(t) = \sigma / < N(t) >$), which has minimum shifted to right from t_m too.

The function P_a has the range of the parameter $\approx 5\sigma$, and form like P_{Nm} but maximum and asymmetry are revealed slightly. The function P_b has the range of the parameter as the P_{Nm}, the mean value is shifted to left from middle and maximum is shifted a few to left from mean value. The form of P_b is co-ordinate with form of P_{Nm}. The function P_c for the parameter $c = N\sqrt{b}$ has form like P_{Nm} but domain of definition, maximum, dispersion and asymmetry are reduced by parameter b. Relative variation of c is less then an another parameter. Since deviation decrease, the function is the most sensibility for accuracy of calculations.

From formal point of view each of the probability density functions is one-maximum function but their forms are variety: from exponential to Gaussian one just as form (1). The general characteristic of the functions is their asymmetry. Experience of the calculation shows the mean values of the function are stability with statistic accuracy, dispersions are stability with accuracy about 5% but interparameter coefficients of correlation (second moment as the dispersion) can change sign if its are less than 10%. The values of high moments and estimates based on them must be used as quality only, but it is insufficient to use ordinary (simple) dispersion. Thus, we must present the distribution function by mean value $\langle\rangle$ and dispersion σ^2 in complex form: as sum of the left and right parts ($\sigma^2 = \sigma_l^2 + \sigma_r^2$) for account of asymmetry the distribution function:
$\int_\alpha^\beta (x- < x >)^2 f(x)dx = \int_\alpha^{<x>} (x- < x >)^2 f(x)dx + \int_{<x>}^\beta (x- < x >)^2 f(x)dx$
where $< x >= \int_\alpha^\beta x f(x)dx$.

These parameters depend weakly on an angle (about 5%) and strongly follows on an energy. The means and the dispersions in the complex fom for zenith angle $\theta = 60^\circ$ can be approximated (accuracy $\approx 2\%$) as:
$t \quad <>= 51 \times \lg_{10} E + 741 \ \sigma_l^2 = -130 \times \lg_{10} E + 744 \ \sigma_r^2 = -256.5 \times lg_{10} E + 1785$,
$N_m \quad <>= 7.69 \times 10^8 \times E^{0.98} \quad \sigma_l^2 = 2.00 \times 10^{14} \times E^{1.51} \quad \sigma_r^2 = 8.88 \times 10^{12} \times E^{1.99}$,
$a \quad <>= 46.4 \times E^{0.093} \quad \sigma_l^2 = 0.187 \times E^{0.523} \quad \sigma_r^2 = 0.160 \times E^{0.549}$,
$b \quad <>= 88872 \times E^{0.054} \quad \sigma_l^2 = 778390 \times E^{0.0773} \quad \sigma_r^2 = 1.39 \times 10^7 \times E^{-0.346}$.

For the parameter $c = N_m\sqrt{b}$ we have similar formulae:
$c \quad <>= 2.29 \times 10^{11} \times E^{1.00736} \ \sigma_l^2 = 6.108 \times 10^{16} \times E^{2.389} \ \sigma_r^2 = 8.067 \times 10^{16} \times E^{2.387}$,
and admissible linear approximation $< c >= 2.40 \times 10^{11} \times E - 1.44 \times 1011$.

The main conclusion is wide range of natural (no statistical) fluctuations of the parameters and variability of form of their distributions. Fore example, as soon as P_{tm} has big asymmetry the most of the experimental data must be below than the mean values but may be any values t_m are placed more higher ones, therefore a description of the parameter by mean value and simple dispersion is not sufficient. The extrapolational supposition at change of $N(t)$ form may has physical explanation by influence of Landau-Pomeranchuk effect as soon as with increase of primary energy the electron-photon cascades develop in more density atmosphere.

Note also, that a small variation of a simulation (which influential for individual cascades is difficult to see) is noticeable for the distribution functions. The ordinary numerical analyze of the distribution functions enable to clear out showers which characteristics is suspicious. Special investigation of the showers ($\approx 0.5\%$ of the statistic) shows some shortcomings of the model and technical of the simulation and way to correct them. In this connection the parameter c is the most useful: "jump-out" point of the distribution and known interpretation of other parameters enable to find, to localize and to check the simulation.

The above presented information assume various representations by approximating functions and for example by form (1). (One can obtain values of the parameters of form (1) as functions $<>$, σ_l^2, σ_r^2.) If the distributions are presented by histograms, one can use methodic of approximation by form (1) presented in [4], but it is needed to note that the distribution functions are not so simple and widthes of bins should be defined carefully (see circles in fig. 1). It is needed addition efforts (mainly mathematical character) for accurate approximating the distribution functions.

3. Conclusion

Note two impotent characteristics the distribution functions of physical parameters of giant showers: 1. Wide range their domain of definition (range of the parameters): $(5 \div 10)\sigma$, 2. Asymmetry. Therefore for identification of showers is needed polyparameter description by distribution functions. The methodic of quit accuracy approximation of distribution functions of the parameters is needed.

4. References

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