Nucleon Decay Matrix Elements on the Lattice

Yasumichi Aoki

RIKEN BNL Research Center

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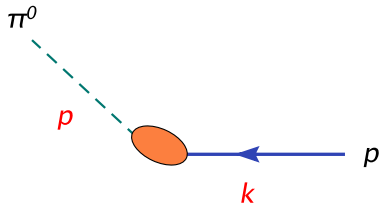
Results obtained in collaboration:

- $N_f = 0, 2$: RBC (RIKEN-BNL-Columbia) collaboration: YA, Dawson, Noaki, Soni, PRD 75 (2007) 014507.
- $N_f = 3$: RBC/UKQCD collaborations in progress: YA, Boyle, Cooney, Dawson, Del Debbio, Izubuchi, Lichtl, Soni, Tweedie.

What is calculated

- $\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{\not{B}}$
- $\mathcal{L}_{\not{B}} = C[\mu] \cdot (qq)(ql)[\mu]$
- $(qq)_{L/R}(lq)_{L/R} = \epsilon^{ijk}(\overline{q^c}^i P_{L/R} q^j)(\overline{l^c} P_{L/R} q^k)$
- $A = C[\mu] \cdot \langle e^+, \pi^0 | (ud)(eu)[\mu] | p \rangle$: example $p \to (\pi^0, e^+)$
- Hadronic part: $\langle \pi^0 | (ud)u[\mu] | p \rangle$ is non-perturbative object, thus needs help of lattice QCD.
- $\mu \simeq 1$ GeV.
- 2 tasks for lattice
  - $(ud)u^{ren}[\mu] = Z[\mu] \cdot (ud)u^{bare}$
  - $\langle \pi^0 | (ud)u^{bare} | p \rangle$

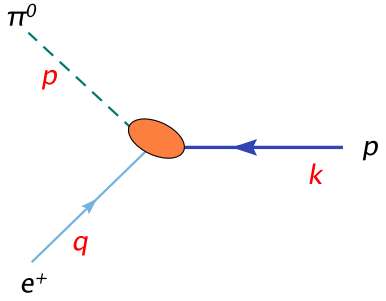
The Relevant Form Factor



$q = k - p$: momentum transfer

$$\langle \pi^0; \vec{p} | (ud) u_L | p; \vec{k}, s \rangle = P_L [W_0(q^2) - \frac{i\not{q}}{m_p} W_q(q^2)] u_p(\vec{k}, s)$$

- W_0 / W_q : relevant / irrelevant form factor [JLQCD].



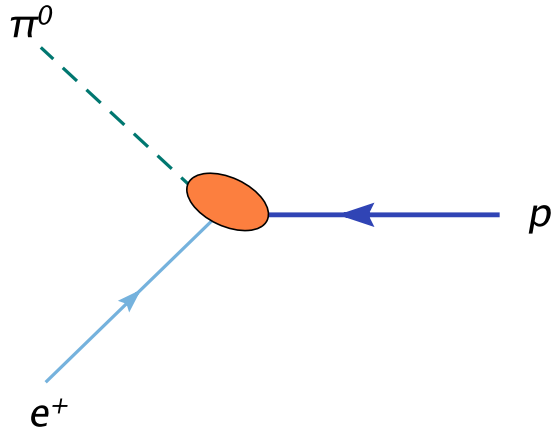
q : momentum of e^+

$$\begin{aligned} \langle e^+; \vec{q}, s' | \langle \pi^0; \vec{p} | (ud)(eu)_L | p; \vec{k}, s \rangle &= W_0 \overline{v_e^c} P_L u_p + W_q \overline{v_e^c} \frac{-i\not{q}}{m_p} P_R u_p \\ &= W_0 \cdot (v_e, u_p)_L + \frac{m_e}{m_p} W_q \cdot (v_e, u_p)_R \end{aligned}$$

- $W_q \simeq W_0$ from ChPT, lattice. W_q term negligible.

Partial Width

written in terms of Wilson Coefficient and the relevant form factor W_0



$$\Gamma(p \to \pi^0 + e^+) = \frac{m_p}{32\pi^2}\left[1 - \left(\frac{m_\pi}{m_p}\right)^2\right]^2 \left|\sum_i C^i W_0{}^i(p \to \pi^0)\right|^2,$$

$$\mathcal{L}_{\not{B}} = \sum_i C^i[\mu] \cdot \mathcal{O}^i[\mu]$$

- W_0
  - depends on operator
  - depends on initial and final state
  - for initial nucleon state, we can calculate W_0 for all the possible qqq operators and final PS states.

Hadronic Matrix Elements

- Parity invariance of (lattice) QCD yields:

$$\begin{aligned}\langle PS;\vec{p}|(qq)_R q_R|N;\vec{k},s\rangle &= \gamma_4\langle PS;-\vec{p}|(qq)_L q_L|N;-\vec{k},s\rangle,\\ \langle PS;\vec{p}|(qq)_L q_R|N;\vec{k},s\rangle &= \gamma_4\langle PS;-\vec{p}|(qq)_R q_L|N;-\vec{k},s\rangle.\end{aligned}$$

- By $m_N \geq m_{PS}$ & $\Delta S \leq 0$, $(p,n) \to (\pi^{0,\pm}, K^{+,0}, \eta)$. All possible $N \to PS$ matrix elements. Isospin symmetry ($u \leftrightarrow d$) reduces the number of MEs.

$$\begin{aligned}\langle\pi^0|(ud)_{R/L}u_L|p\rangle &= \langle\pi^0|(du)_{R/L}d_L|n\rangle,\\ \langle\pi^+|(ud)_{R/L}d_L|p\rangle &= -\langle\pi^-|(du)_{R/L}u_L|n\rangle,\\ \langle K^0|(us)_{R/L}u_L|p\rangle &= -\langle K^+|(ds)_{R/L}d_L|n\rangle,\\ \langle K^+|(us)_{R/L}d_L|p\rangle &= -\langle K^0|(ds)_{R/L}u_L|n\rangle,\\ \langle K^+|(ud)_{R/L}s_L|p\rangle &= -\langle K^0|(du)_{R/L}s_L|n\rangle,\\ \langle K^+|(ds)_{R/L}u_L|p\rangle &= -\langle K^0|(us)_{R/L}d_L|n\rangle,\\ \langle\eta|(ud)_{R/L}u_L|p\rangle &= -\langle\eta|(du)_{R/L}d_L|n\rangle.\end{aligned}$$

- isospin limit

$$\langle\pi^+|(ud)d|p\rangle = \sqrt{2}\langle\pi^0|(ud)u|p\rangle$$

How to calculate W_0

- direct method: direct measurement through 3- and 2-point functions
- indirect method: Chiral Perturbation Theory + Low Energy Constants evaluated on the lattice
  - $\mathcal{L}_B^\chi(f, D, F, , ,) + \mathcal{L}_{\not{B}}^\chi(\alpha, \beta)$, $\quad (D + F = g_A)$
  - On the lattice, measure α and β

$$\langle 0|(ud)_R u_L|p\rangle = \alpha P_L u_p,$$

$$\langle 0|(ud)_L u_L|p\rangle = \beta P_L u_p$$

  - Reduction formula for $p \to \pi^0$:

$$W_0(\langle \pi^0|(ud)_R u_L|p\rangle) = \alpha(1 + D + F)/\sqrt{2}f,$$

$$W_0(\langle \pi^0|(ud)_L u_L|p\rangle) = \beta(1 + D + F)/\sqrt{2}f$$

  - Reduction formula available for all the matrix elements [Claudson-Wise-Hall, JLQCD].
  - These are only the lowest order approximations. As the pion has large momentum $|\vec{p}| \simeq m_p/2$, the applicability is questionable.

Systematic Errors

Potential systematic uncertainties in lattice calculations:

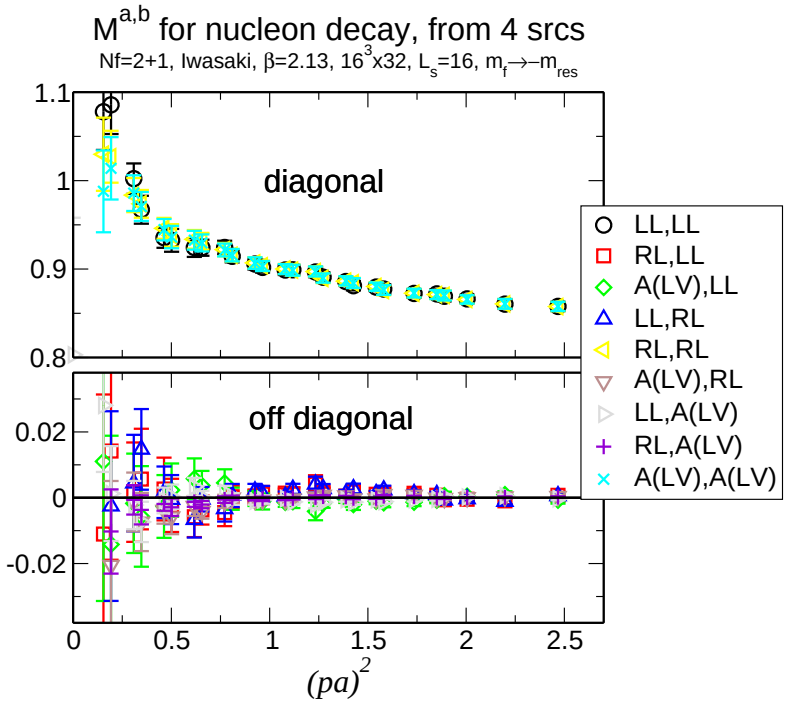
- Systematic error of indirect methods:
  - It is an approximation. Size of the error unknown.
- Systematic errors for both direct and indirect methods:
  - Lattice perturbation theory has far worse convergence than the continuum perturbation.
  - Finite lattice spacing a.
  - Finite physical volume.
  - Quenching:
    - ★ $N_f = 0$: quenched approximation neglects vacuum polarization of all quarks. Theory is non unitary.
    - ★ $N_f = 2$: taking into account vacuum polarization of u, d quarks. s quark is quenched.
    - ★ $N_f = 3$: taking into account vacuum polarization of u, d s quarks, thus free from quenching error.

To reduce systematic errors

- Solutions
  - Use direct method.
  - Use non-perturbative renormalization.
  - Use lattice action which has the properties of continuum action as much as possible. Take the continuum limit $a \to 0$ or investigate a dependence.
  - Investigate finite volume dependence.
  - Perform $N_f = 3$ simulation.
    - ★ State-of-the-art supercomputer and recent algorithmic innovation made it possible to simulate $N_f = 3$ even with our expensive fermions.
- Our lattice action
  - We use domain-wall fermions (DWF) for quarks.
  - The lattice exact chiral symmetry is realized at $L_s \to \infty$, where L_s is the size of 5th dimension.
  - We use $L_s = 16$, which induces the explicit chiral symmetry breaking. But it is negligible in practice.
  - Automatically $\mathcal{O}(a)$ improved: leading scaling violation absent

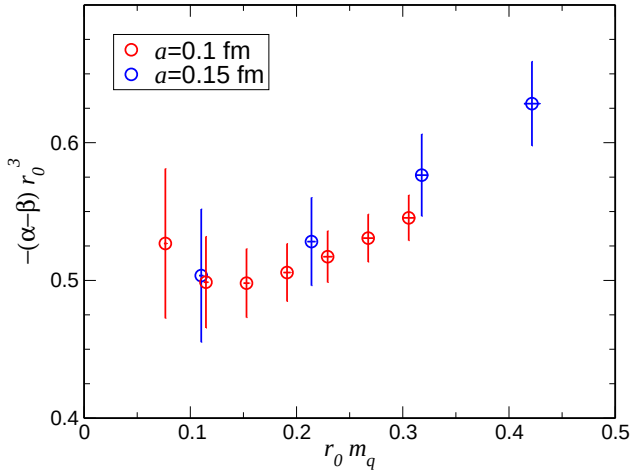
Some checks

Non-perturbative $Z_q^{3/2}/Z_{ND}$ for $N_f = 3$



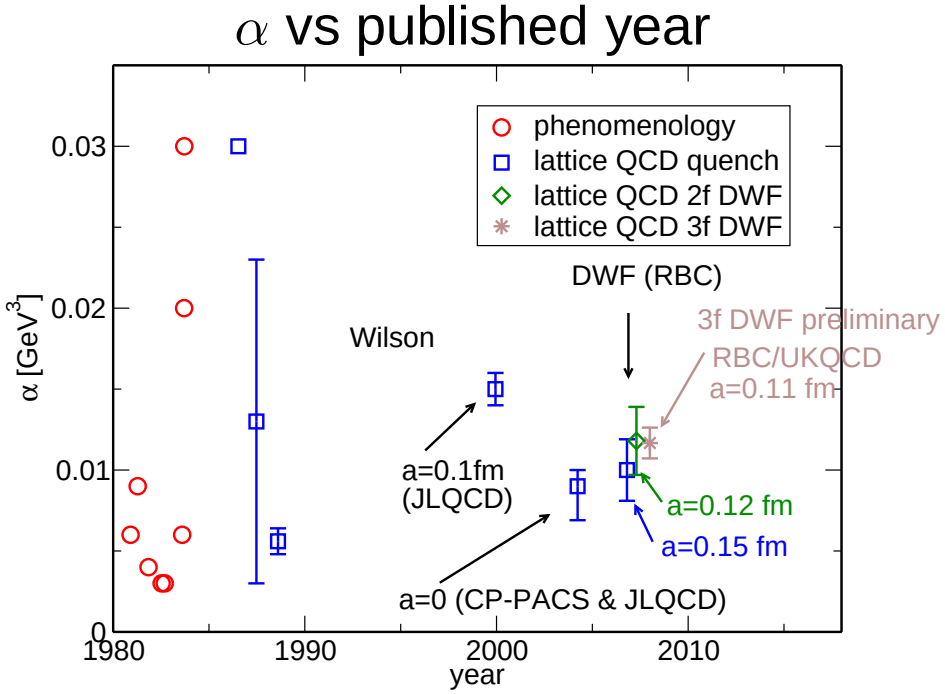
- Off-diagonal elements are consistent with zero, which is expected from the exact chiral symmetry.

Finite a and V effect of $\alpha - \beta$ for $N_f = 0$



- Note: $\beta \simeq -\alpha$.
- Blue: $L = 2.4$ fm. We will use.
- Red: $L = 1.6$ fm.
- Finite a and V effects are negligible.

Low energy constants summary



- Matched to $\overline{\text{MS}}$, NDR, $\mu = 2$ GeV at NLO.
- $N_f = 3$ result is still preliminary.
- $\alpha = 0.01$ GeV^3 may be a representative value.
- $\beta \simeq -\alpha$.
- But the story does not end....

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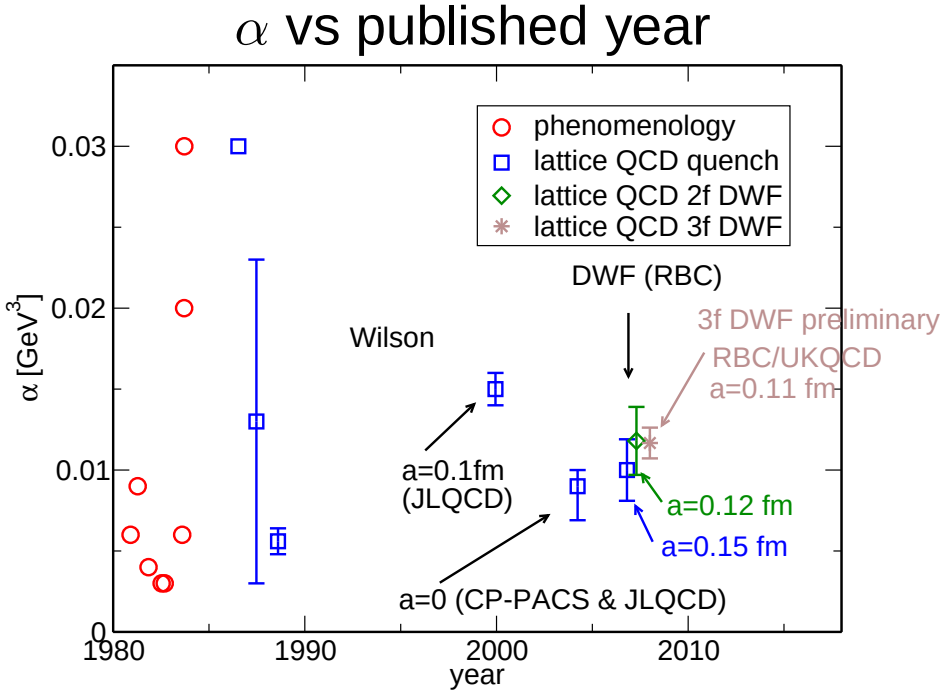

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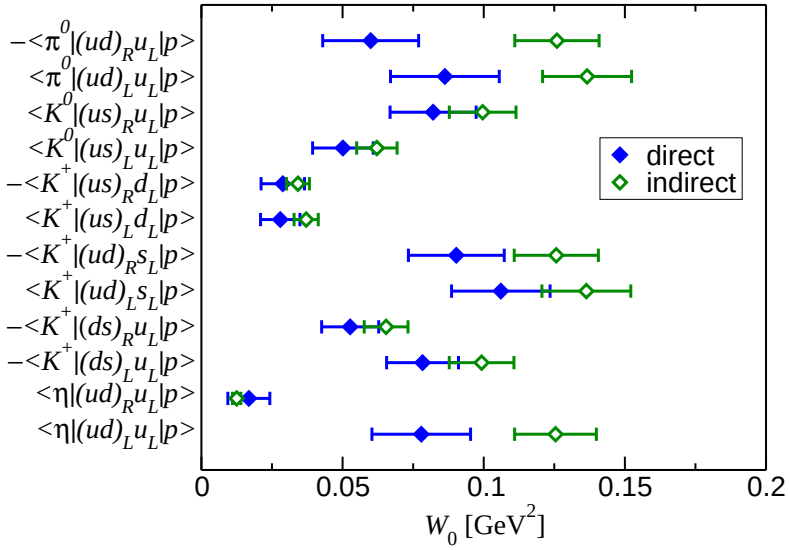
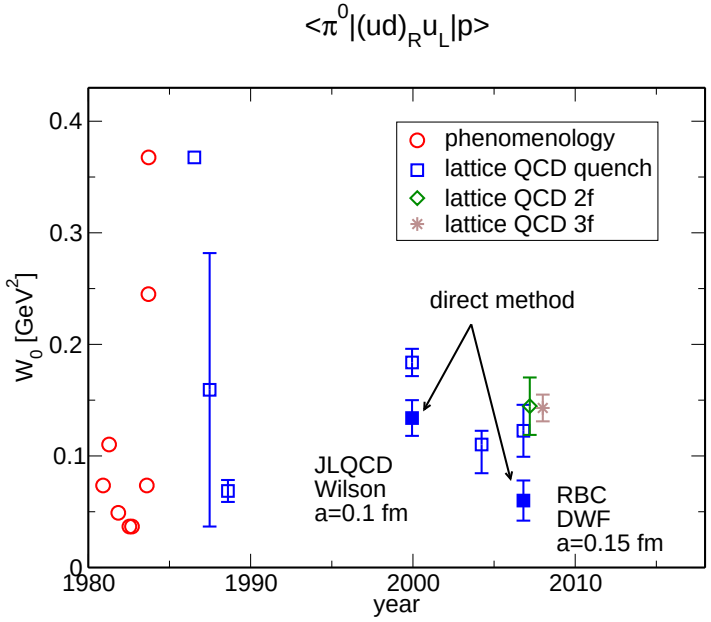
W_0 summary

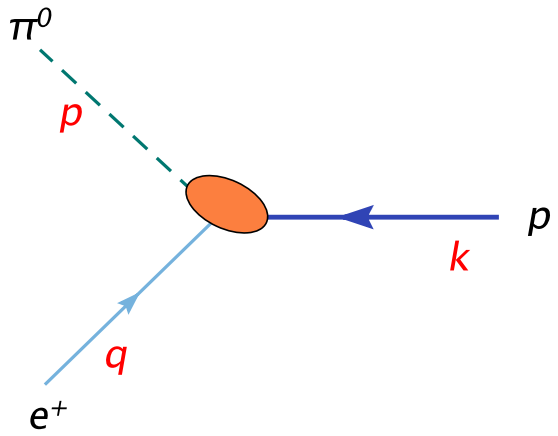
$N_f = 0$, $a = 0.15$ fm

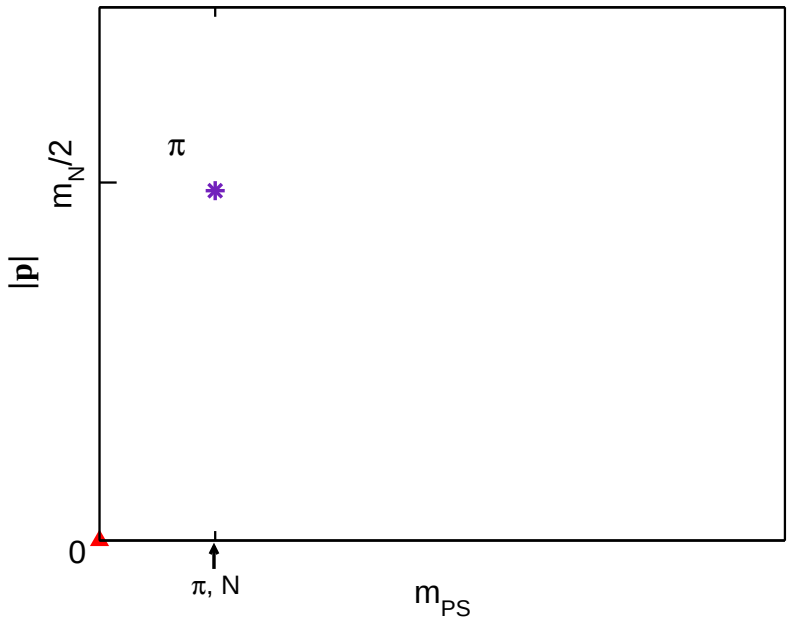




- Similar pattern has been observed by JLQCD with $a = 0.1$ fm, Wilson Fermion. Direct-indirect diff is larger here.
- Indirect method tends to overestimate W_0, results in underestimating proton lifetime. Note: $\tau \propto 1/W_0^2$
- Largest difference observed for $p \to \pi$ decay.
  - $\tau(\text{direct})/\tau(\text{indirect}) = 2 - 4$.
- (If you do not care a factor 4 for the proton lifetime, you can safely use indirect results.)

Something's wrong ?





Direct method is preferable.

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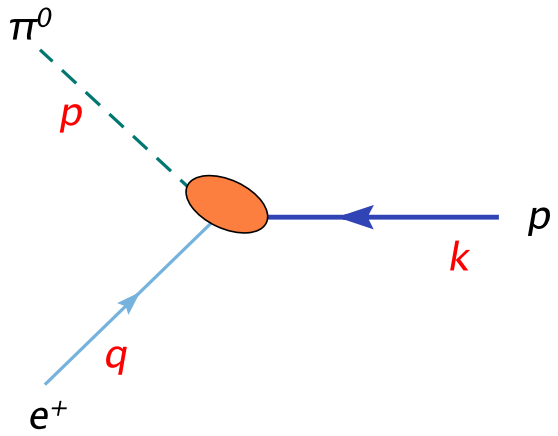

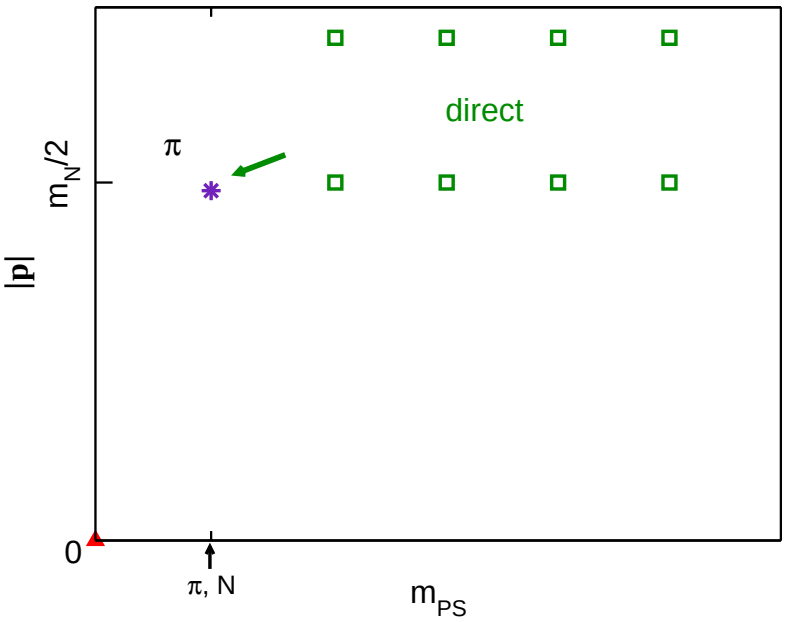

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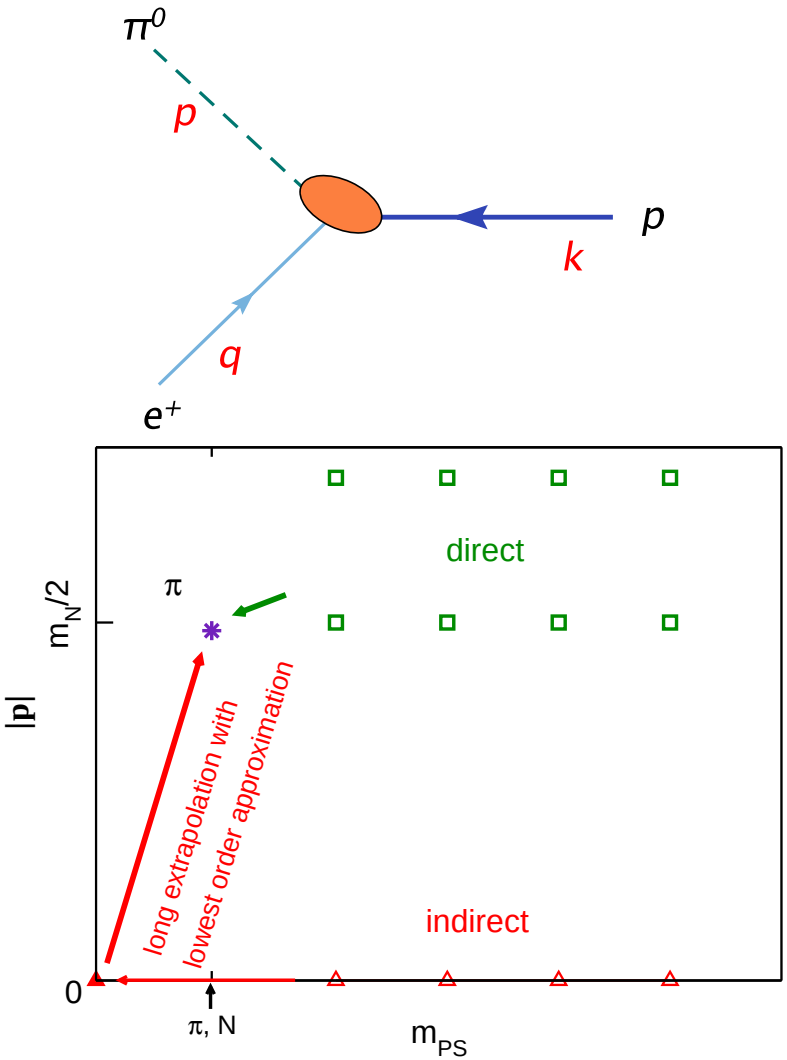

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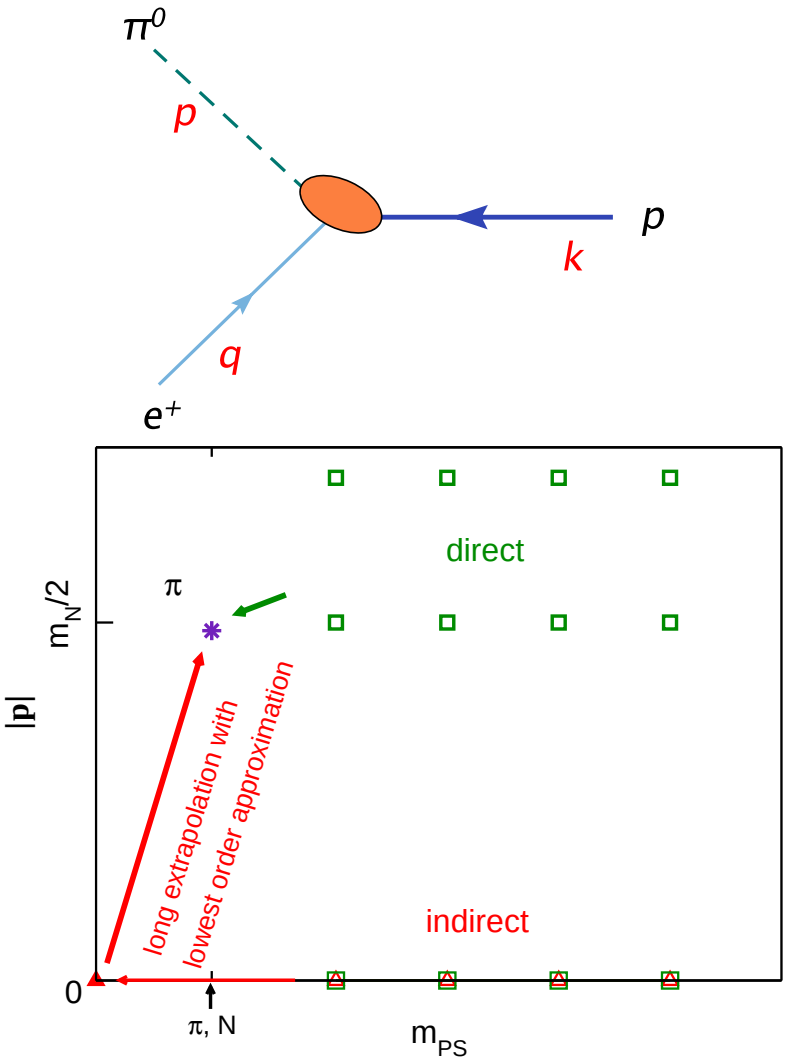
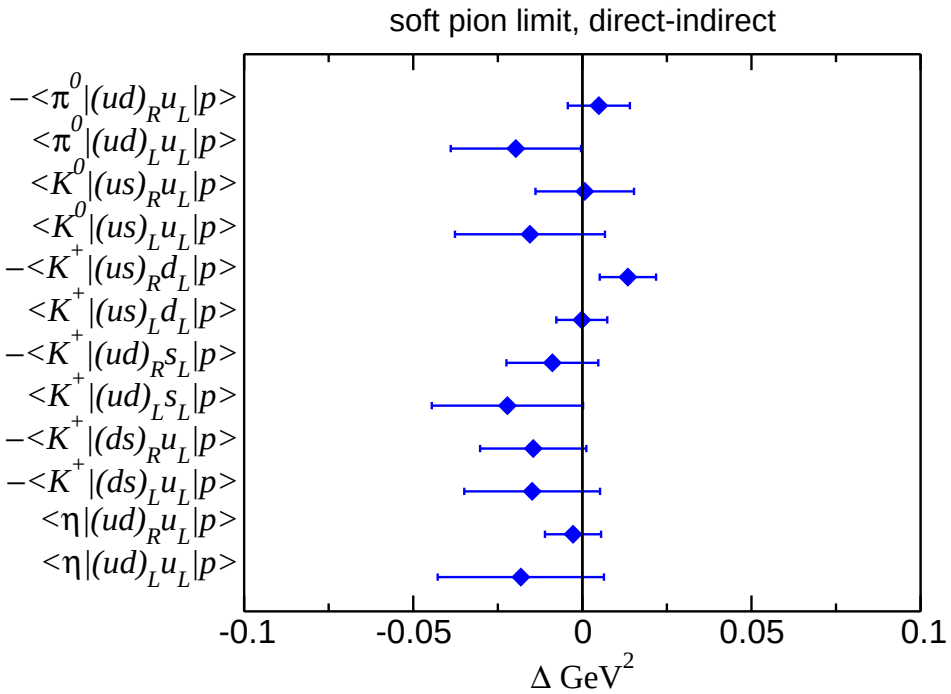


Direct method is preferable.

$$W^{\text{direct}} - W^{\text{indirect}}(\alpha, \beta)$$

must vanish in the soft pion limit



- Soft pion theorem is OK!
- Nothing is wrong!

Summary and Outlook

- Fundamental quantity is W_0.
- For the indirect estimate of W_0, α and β has been calculated for $N_f = 0$ and 2, and is being calculated for $N_f = 3$.
  - Preliminary $N_f = 3$ results are consistent with $N_f = 0$ and 2.
- Direct estimate of W_0 for all the possible states and operators have been obtained for $N_f = 0$.

  ($SU(3)$ flavor breaking effects have not been taken into account for η).
- Indirect method tends to overestimate W_0 for $N_f = 0$, underestimating proton lifetime.
  - We need to do direct measurement of W_0 for $N_f = 3$, which is in preparation !
- Remaining uncertainty is a dependence,
  - which is expected to be small from the experience of $N_f = 0$. This should be confirmed in future study.